Effects of Zonal Flows on Equatorially Trapped Waves

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ABSTRACT

Linear shallow water equations are employed to study the effects of basic zonal flows on equatorially trapped waves. Analytical solutions are obtained for constant basic zonal flows. It is shown that changes in the wave structures due to the non-Doppler effect of the basic zonal flow are considerable for the Rossby wave, moderate for the westward propagating mixed Rossby-gravity wave, but negligible for the other equatorial waves. The Rossby wave and the westward propagating mixed Rossby-gravity wave possess greater eigenfrequencies and are less trapped in westerlies than in easterlies. The dependence of the Rossby wave structure upon the basic zonal flow is interpreted in terms of potential vorticity conservation. In basic zonal flows with meridional shears, the eigenfrequencies are found to be larger in equatorial easterlies than in equatorial westerlies for the westward propagating waves but smaller for the eastward propagating waves. While the meridional structures of the eastward propagating waves show little sensitivity to the basic zonal flow, the Rossby wave is less trapped in equatorial westerlies but the westward propagating inertia-gravity wave is less trapped in equatorial easterlies. It is suggested, therefore, that the equatorial transient Rossby waves interact with midlatitudes more readily at the longitudes associated with tropical westerlies. Furthermore, at these same longitudes, it is possible that extratropical forcing may project onto the equatorial modes and produce equatorially trapped responses if the forcing lies within their turning latitudes, which may extend to beyond 40° latitude at these locations. The conclusion underlines the upper troposphere of the tropical eastern Pacific Ocean and possibly the tropical Atlantic Ocean as critical regions of latitudinal interaction in both directions over a wide range of time scales.

1. Introduction

During the last two decades, the theory of equatorial waves has become the cornerstone of equatorial dynamics in the atmosphere. Matsuno (1966) derived the complete set of linear wave solutions of the shallow water equations on an equatorial β-plane and noted the trapped properties of these waves. Longuet-Higgins (1968) pointed out that Matsuno's solutions are asymptotic approximations of general normal modes on a sphere as the parameter $\epsilon = 4\Omega^2 a^2/gh$ approaches infinity. Here Ω is the angular velocity of the earth's rotation, a the earth's mean radius, g the gravitational acceleration, and h the equivalent depth of a shallow fluid. The equatorially trapped waves have been applied for various purposes, especially in explaining some fundamental features of tropical climate. The phenomena that have been explained by using the equatorial wave theory include the Walker circulation (Webster 1972, 1973; Gill 1980; Lim and Chang 1983), the atmospheric teleconnection patterns (Lim and Chang 1983; Lau and Lim 1984), the low-frequency oscillation initially observed by Madden and Julian (1971) (e.g., Chao 1987; Lau and Peng 1987; Wang and Rui 1989), and the El Niño–Southern Oscillation (e.g., Lau 1981; Hirst 1986).

In the applications of the equatorial wave theories noted above, the impacts of the atmospheric basic state on the structures of equatorial waves need to be well understood and have been the subject of a number of studies. Using a linear shallow water equation model, Lim and Chang (1983) demonstrated that steady state Rossby wave responses to an equatorial forcing are much less trapped in a constant basic westerly flow than in an easterly flow. Lau and Lim (1984) also found that the equatorially forced Rossby wave is able to "radiate" toward higher latitudes only within a westerly wind regime. Boyd (1978a,b) obtained asymptotic equatorial wave solutions in a mean zonal flow with meridional shear. He showed that, while the shear effects are negligible for the structures of the Kelvin and Rossby waves, they are considerable for the mixed Rossby–gravity wave. Wilson and Mak (1984) found that a meridionally sheared zonal flow can give rise to equatorial trapping. Lau and Lim (1984) suggested that a basic westerly shear zonal flow would focus energy associated with the equatorial Rossby mode from the tropics into the extratropics more readily than the case with zero or easterly shear. The temporal behavior of the equatorial Rossby wave in a sheared flow has been

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studied by Boyd and Christidis (1987). The structures and vertical propagating properties of the equatorial waves in a mean flow with vertical shear have also received considerable attention (e.g., Lindzen 1971, 1972).

Recently, Webster and Chang (1988) demonstrated that in the linear regime, energy associated with the transient equatorial Rossby wave excited in a zonally nonuniform basic flow may propagate along the equator and accumulate in a region to the east of tropical westerly maximum. From the accumulation region, where the characteristics of the basic state and the wave conspire to provide a convergence of wave action flux, the signal appears to emanate toward higher latitudes. Their analysis has now been extended to the nonlinear regime where essentially the same features are evident (Chang and Webster 1989). A central feature of Webster and Chang's wave energy accumulation–emanation theory is that irrespective of where the waves are generated along the equator, emanation of energy to higher latitudes occurs at the same longitudes, especially where the longitudinal stretching deformation of the basic zonal flow is negative. The similarity of the response may explain results such as those from Geisler et al. (1985), where it was found that the middle latitude atmospheric response seems to be relatively insensitive to the location of forcing along the equator and appears to be geographically phase-locked to the longitudes associated with the equatorial westerlies. The mechanism for the energy emanation, however, is unknown. Indeed, if it is a true emanation at all, why would wave energy not emanate directly towards higher latitude away from the wave source within the tropical easterlies as suggested initially by Hoskins and Karoly (1981)? Based on existing theory, there does not appear to be any reason to expect an independence of the extratropical response to the location of the equatorial forcing. Another problem with the wave-train emanation interpretation is that the initial waves generated in the easterlies were highly equatorially trapped and, by definition, possessed collectively zero poleward group velocities. If the waves in the westerlies were truly emanating, they would have nonzero poleward group velocities. The production of this new wave form in the accumulation region would require a wave–wave interaction. Obviously, such nonlinear processes were absent from Webster and Chang's initial linear analysis. The wave energy emanation mechanism, therefore, must reside within linear theory.

In addition to theories concerning equatorial waves, there has been considerable evidence of their existence from observations. The waves that have been observed include Kelvin waves in the stratosphere (e.g., Wallace and Kousky 1968) and troposphere (e.g., Zanvil and Yanai 1980; Yanai and Lu 1983; Lu 1987), westward propagating mixed Rossby-gravity waves in the stratosphere (e.g., Yanai and Maruyama 1966; Wallace 1973) and troposphere (e.g., Zanvil and Yanai 1980; Yanai and Lu 1983; Lu 1987; Liebmann and Hendon 1989), and Rossby waves in the troposphere (e.g., Reed and Recker 1971; Zanvil and Yanai 1980; Yanai and Lu 1983). A common feature is that the characteristics of the observed equatorial waves appear to vary in time and space. For example, in Yanai and Lu (1983), signals of the mixed Rossby–gravity waves are strong in the 200 mb wind field data in the summer of 1967, but are absent from the data in the summer of 1972. Furthermore, the periods and equivalent depths of the observed Rossby waves from the two summers are different. Liebmann and Hendon (1989) recently observed some zonal variations in the characteristics of the westward propagating mixed Rossby–gravity waves in the lower troposphere. They found larger frequencies and smaller zonal scales over the Indian and western Pacific oceans than over the other equatorial longitudes. The reasons for the observed temporal and zonal variabilities associated with the wave characteristics are not clear. They are probably due to the variations of wave excitation mechanisms or the dependence of the waves themselves upon the atmospheric basic state that varies temporally and zonally, or a combination of both factors.

In order to tackle the questions arising from the observations and theoretical studies in particular, and to gain a better understanding of tropical atmospheric dynamics in general, it is necessary to investigate further the effects of the varying basic states upon the equatorially trapped waves in the atmosphere.

The purpose of this study is to examine the effects of basic state zonal flows on the equatorially trapped waves. In sections 2 and 3, equatorial wave solutions in constant basic zonal flows are obtained from a set of linear shallow water equations on an equatorial β-plane. It will be shown that, while the Kelvin wave, inertia-gravity wave, and eastward propagating mixed Rossby–gravity wave are not particularly sensitive to the basic zonal flows, the frequencies and structures of the Rossby wave and westward propagating mixed Rossby–gravity wave are subject to considerable modulation by non-Doppler effects of the basic zonal flows. In a constant basic westerly flow, the eigenfrequencies of the Rossby wave and westward propagating mixed Rossby–gravity wave are larger and the meridional structures of these waves less trapped to the equator than in a constant basic easterly flow. In section 4, the non-Doppler effect of the basic zonal flow on the Rossby wave trapping is interpreted in terms of potential vorticity conservation. Solutions of equatorial waves in sheared basic flows will be obtained in section 5. It will be shown that larger frequencies are found in the basic zonal flow of equatorial easterlies with strong shear than equatorial westerlies with weak shear for the westward propagating waves. The opposite results are obtained for the eastward propagating waves. In basic zonal flows with equatorial easterlies, the Rossby wave shows more trapped structure while the westward inertia–gravity wave exhibits less trapped structures than in basic zonal flows with equatorial westerlies. The meridional structures of the eastward propagating waves seem to be insensitive to the basic zonal flow, whether shear exists or not. In section 6, the results are summarized and conclusions drawn regarding the implications of the sensitivities of equatorial waves on the basic zonal flow. In particular, it is emphasized that the regions of upper tropospheric westerlies over the equatorial West Pacific and Atlantic oceans may act as two-way corridors of interactions between the tropics and extratropics.

2. Equations

The equatorially trapped waves have been described with the shallow water equations on an equatorial β-plane in some detail by Matsuno (1966). Here we consider a more general system with a shear basic state zonal flow, $U(y)$. The total geopotential field of such a system can be expressed as

$$\Phi(x, y, t) = \Phi_0 + \Phi_s(y) + \phi(x, y, t), \quad (2.1)$$

where $\phi(x, y, t)$ is the geopotential perturbation and $\Phi_s(y)$ the geopotential surface slope which is in geostrophic balance with the basic zonal flow $U(y)$, such that

$$\frac{d\Phi_s}{dy} = -\beta y U. \quad (2.2)$$

In (2.1) Φ_0 is the constant mean geopotential field and is related with the equivalent depth h as $\Phi_0 = gh$. In this study, we consider a system where $\Phi_0 \gg \Phi_s(y)$.

The linear, inviscid, shallow water equations for the system on an equatorial β-plane are

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + v\frac{dU}{dy} - \beta y v + \frac{\partial \phi}{\partial x} = 0, \quad (2.3a)$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + \beta y u + \frac{\partial \phi}{\partial y} = 0, \quad (2.3b)$$

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} - \beta y U v + gh\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \quad (2.3c)$$

where u and v are the zonal and meridional velocity components and ϕ the geopotential field. Here (2.3) can be made nondimensional by choosing time and length scales of

$$T = \beta^{-1/2}(gh)^{-1/4}, \quad L = \beta^{-1/2}(gh)^{1/4}. \quad (2.4)$$

The equations for the nondimensional variables are

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + v\frac{dU}{dy} - yv + \frac{\partial \phi}{\partial x} = 0, \quad (2.5a)$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + yu + \frac{\partial \phi}{\partial y} = 0, \quad (2.5b)$$

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} - yUv + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.5c)$$

Bennett and Young (1971) employed the same set to study meridional wave propagation. The equivalent depth h used in this study for nondimensionalization is 300 m, which is close to one of the calculated values from the observations of the equatorially trapped Rossby wave (Yanai and Lu 1983). Notice that the terms in (2.5) with the basic zonal flow can be distinguished as those of a Doppler effect (i.e., $U\partial/\partial x$) and a non-Doppler effect (i.e., $v\,dU/dy$ and $-yUv$).

We seek the normal mode solutions of (2.5), i.e.,

$$\begin{pmatrix} u \\ v \\ \phi \end{pmatrix} = \begin{pmatrix} u'(y) \\ v'(y) \\ \phi'(y) \end{pmatrix} \exp[i(kx - \omega t)], \quad (2.6)$$

where k is the zonal wavenumber, and ω the frequency; both being assumed constant. Substituting (2.6) into (2.5) and omitting the primes for simplicity, we obtain the equations for the meridional eigenfunctions as

$$-i\hat{\omega}u - yv + ik\phi + \frac{dU}{dy}v = 0, \quad (2.7a)$$

$$-i\hat{\omega}v + yu + \frac{d\phi}{dy} = 0, \quad (2.7b)$$

$$-i\hat{\omega}\phi + iku + \frac{dv}{dy} - yUv = 0, \quad (2.7c)$$

where $\hat{\omega}$ is the Doppler-shifted frequency, which will be determined as the eigenvalue in sections 3 and 5. Here $\hat{\omega}$ is related to ω by the Doppler relation

$$\hat{\omega} = \omega - kU. \quad (2.8)$$

A single equation for $v(y)$ can be derived from (2.7) by eliminating u and ϕ. This is

$$\frac{d^2v}{dy^2} + \frac{dv}{dy}\left(\frac{2\hat{\omega}k}{\hat{\omega}^2 - k^2}\frac{dU}{dy} - Uy\right) + v\left[\hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} - y^2 - \frac{k}{\hat{\omega}}Uy^2 - U + \frac{2k^2}{\hat{\omega}^2 - k^2}\left(\frac{dU}{dy}\right)^2 - \frac{2k^2}{\hat{\omega}^2 - k^2}y\frac{dU}{dy} + \frac{k}{\hat{\omega}}\frac{d^2U}{dy^2} - \frac{k}{\hat{\omega}}\frac{\hat{\omega}^2 + k^2}{\hat{\omega}^2 - k^2}yU\frac{dU}{dy}\right] = 0. \quad (2.9)$$

The boundary condition for (2.9) is

$$v(y) \to 0 \quad \text{as} \quad y \to \pm\infty, \quad (2.10)$$

which is an approximation of the homogeneous boundary condition on a sphere (i.e., $v = 0$ at the poles) and is necessary for $v(y)$ on an equatorial β-plane to be a valid approximation to a solution on a sphere (Lindzen 1967).

Letting

$$p(y) = \frac{2\hat{\omega}k}{\hat{\omega}^2 - k^2}\frac{dU}{dy} - Uy, \quad (2.11a)$$

$$q(y) = \hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} - y^2 - \frac{k}{\hat{\omega}}Uy^2 - U + \frac{2k^2}{\hat{\omega}^2 - k^2}\left(\frac{dU}{dy}\right)^2 - \frac{2k^2}{\hat{\omega}^2 - k^2}y\frac{dU}{dy} + \frac{k}{\hat{\omega}}\frac{d^2U}{dy^2} - \frac{k}{\hat{\omega}}\frac{\hat{\omega}^2 + k^2}{\hat{\omega}^2 - k^2}yU\frac{dU}{dy}, \quad (2.11b)$$

(2.9) can be written as

$$\frac{d^2v}{dy^2} + p(y)\frac{dv}{dy} + q(y)v = 0. \quad (2.12)$$

By applying the variable transformation

$$v(y) = V(y)\exp\left[-\int^y \tfrac{1}{2}p(\zeta)d\zeta\right], \quad (2.13)$$

(2.12) becomes

$$\frac{d^2V}{dy^2} + V\left(q - \frac{1}{2}\frac{dp}{dy} - \frac{1}{4}p^2\right) = 0, \quad (2.14)$$

and with (2.11), can be written as

$$\frac{d^2V}{dy^2} + V\left\{\hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} - \left[1 + \frac{k}{\hat{\omega}}U + \frac{1}{4}U^2\right]y^2 - \frac{1}{2}U - \frac{3k^4}{\hat{\omega}^2 - k^2}\left(\frac{dU}{dy}\right)^2 - \frac{k^3}{\hat{\omega}(\hat{\omega}^2 - k^2)}\frac{d^2U}{dy^2} + \left[\frac{1}{2} - \frac{k^3U}{\hat{\omega}(\hat{\omega}^2 - k^2)} - \frac{2k^2}{\hat{\omega}^2 - k^2}\right]y\frac{dU}{dy}\right\} = 0. \quad (2.15)$$

A generic form of (2.15) is

$$\frac{d^2V}{dy^2} + \Gamma^2(y)V = 0, \quad (2.16)$$

where $\Gamma^2(y)$, which represents the quantity in the braces of (2.15), is the squared refractive index, and is commonly used in different forms to study propagating and trapping properties of the solution $V(y)$ (e.g., Bennett and Young 1972; Lau and Lim 1984; Wilson and Mak 1984). In general, the solution $V(y)$ will be wavelike where $\Gamma^2(y) > 0$ and evanescent where $\Gamma^2(y) < 0$. Equatorial trapping of $V(y)$, then, requires that

$$\begin{cases} \Gamma^2 > 0, & \text{when} \quad y < y_t, \\ \Gamma^2 < 0, & \text{when} \quad y > y_t, \end{cases} \quad (2.17)$$

where y_t is a turning latitude, at which $\Gamma^2 = 0$ and the solution changes from being wavelike to evanescent.

For constant U, a simpler form can be derived from (2.15) for which analytical solutions are available as derived in section 3. For a basic zonal flow with shear, an eigenvalue problem can be defined from (2.7) and solutions are obtained as eigenvectors in section 5.

3. Meridional solutions in constant zonal flows

When the basic state zonal flow is constant, (2.15) reduces to

$$\frac{d^2V}{dy^2} + V\left[\hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} - \left(1 + \frac{k}{\hat{\omega}}U\right)y^2\right] = 0, \quad (3.1)$$

where the terms $\frac{1}{4}U^2$ and $\frac{1}{2}U$ have been neglected by assuming $U \ll 1$ (i.e., $U \ll (gh)^{1/2}$ in dimensional form). Noticing that in the squared refractive index

$$\Gamma^2 = \hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} - \left(1 + \frac{k}{\hat{\omega}}U\right)y^2 \quad (3.2)$$

all the parameters are constant, we may write (3.1) in a generic form as

$$\frac{d^2V}{dy^2} + V(a - b^2y^2) = 0, \quad (3.3a)$$

where

$$a = \hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}}, \quad (3.3b)$$

and

$$b = \left(1 + \frac{k}{\hat{\omega}}U\right)^{1/2}. \quad (3.3c)$$

With constant a and b, and the boundary condition (2.10), (3.3a) has the standard solution

$$V(y) = H_n(b^{1/2}y)\exp(-\tfrac{1}{2}by^2), \quad (3.4)$$

where n is the meridional mode number and $H_n(\xi)$ is the Hermite polynomial of the nth order. The dispersion relation associated with (3.4) is

$$a = (2n + 1)b. \quad (3.5)$$

From (3.3a) and (3.5), the turning latitude is given by

$$y_t = \pm\left(\frac{2n + 1}{b}\right)^{1/2}. \quad (3.6)$$

The solution for (2.9) in a constant basic zonal flow is then obtained from (2.13) and (3.4) as

$$v(y) = H_n(b^{1/2}y)\exp\left(-\tfrac{1}{2}by^2 + \tfrac{1}{4}Uy^2\right). \quad (3.7)$$

Substituting (3.3b, c) into (3.5)–(3.7), we have the solution, the associated dispersion relation and the turning latitude as

$$v(y) = H_n\left[\left(1+\frac{k}{\hat{\omega}}U\right)^{1/4} y\right] \times \exp\left\{-\left[\frac{1}{2}\left(1+\frac{k}{\hat{\omega}}U\right)^{1/2} - \frac{1}{4}U\right]y^2\right\}, \quad (3.8)$$

$$\hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} = (2n+1)\left(1+\frac{k}{\hat{\omega}}U\right)^{1/2}, \quad (3.9)$$

$$y_t = \pm\left[(2n+1)\left(1+\frac{k}{\hat{\omega}}U\right)^{1/2}\right]^{1/2}, \quad (3.10)$$

respectively.

We define a meridional structure function as

$$\mathbf{F}_n(y) \equiv \exp\left[-\frac{1}{2}\left(\frac{y}{R}\right)^2\right]H_n\left(\frac{y}{R}\right), \quad (3.11a)$$

where

$$R = \left(1+\frac{k}{\hat{\omega}}U\right)^{-1/4} \quad (3.11b)$$

is the modified equatorial Rossby radius of deformation in a constant basic state zonal flow. Note that, from (3.10) and (3.11b), the turning latitude and the Rossby radius of deformation are related to each other as

$$y_t = \pm(2n+1)^{1/2}R. \quad (3.11c)$$

Consequently, *the two quantities have the same dependence on the constant basic zonal flow!*

If the small term $\frac{1}{4}Uy^2$ in (3.8) is neglected, the complete solutions to the reduced set (2.5), with a constant U, can be expressed as

$$\begin{pmatrix} u \\ v \\ \phi \end{pmatrix} = \begin{pmatrix} [u_1\mathbf{F}_{n+1}(y) + u_2\mathbf{F}_{n-1}(y)]/(iW) \\ \mathbf{F}_n(y) \\ [\phi_1\mathbf{F}_{n+1}(y) + \phi_2\mathbf{F}_{n-1}(y)]/(iW) \end{pmatrix} \times \exp[i(kx-\omega t)] \quad (3.12a)$$

where

$$u_1 = -\frac{1}{2}[(\hat{\omega}+kU)R + k/R], \quad (3.12b)$$

$$u_2 = -n[(\hat{\omega}+kU)R - k/R], \quad (3.12c)$$

$$\phi_1 = -\frac{1}{2}[(\hat{\omega}U+k)R + \hat{\omega}/R], \quad (3.12d)$$

$$\phi_2 = -n[(\hat{\omega}U+k)R - \hat{\omega}/R], \quad (3.12e)$$

and

$$W = \hat{\omega}^2 - k^2. \quad (3.12f)$$

In (3.12), the eigenfrequency $\hat{\omega}$ must be determined from the dispersion relation (3.9) for each wave. As demonstrated by Matsuno (1966) and others, the various equatorial waves need to be addressed separately for they bear different dispersion relations and possess different structures. Figure 1 shows the dispersion relations for these families of waves in zero basic state. We now consider, in turn, the eigenfrequencies and wave structures of each waveform in constant basic zonal flows of 0.18, 0, and −0.18; or, in dimensional form, 10, 0, and −10 m s^{-1}.

a. Rossby wave

From (3.9), the approximate dispersion relation for the Rossby wave is

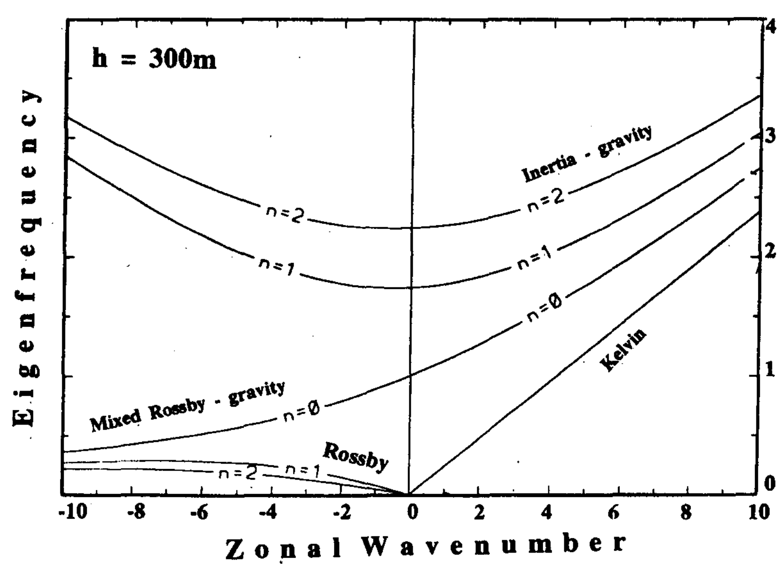

FIG. 1. Dispersion relations of the equatorial waves. Labels refer to the wave's families.

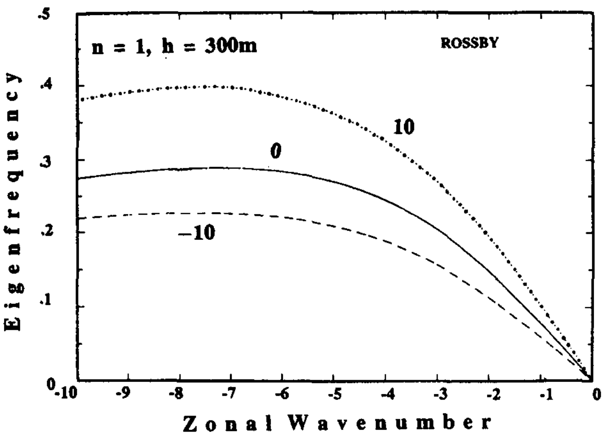

FIG. 2. Dispersion relations of the $n = 1$ Rossby wave with $U = 10$ (dotted line), 0 (solid line), and -10 (dashed line) m s^{-1}.

$$-k^2 - \frac{k}{\hat{\omega}} = (2n+1)\left(1 + \frac{k}{\hat{\omega}}U\right)^{1/2},$$

from which the eigenfrequency is obtained as

$$\hat{\omega} = -\frac{k}{2}\frac{2k^2 - (2n+1)^2U - (2n+1)\Lambda^{1/2}}{k^4 - (2n+1)^2}, \tag{3.13a}$$

where

$$\Lambda = (2n+1)^2U^2 - 4k^2U + 4. \tag{3.13b}$$

The dependence of Rossby wave eigenfrequency upon the basic zonal flow is shown in Fig. 2 where $\hat{\omega}$ from (3.13) is plotted as a function of k for the three constant basic zonal flows with $n = 1$. For a given k and n, a larger $\hat{\omega}$ is found in the westerlies than in the easterlies. This dependence of $\hat{\omega}$ on U is more substantial for larger k. Consequently, the Doppler-shifted phase speed of the Rossby wave, which is westward, is larger in the basic westerlies than in the easterlies.

The group velocity of the Rossby wave can be derived from (3.13) as

$$\hat{C}g = -\frac{1}{2}\frac{1}{k^4 - (2n+1)^2}\Big\{2k^2 - (2n+1)^2U - (2n+1)\Lambda^{1/2} + 4k^2 - 4(2n+1)k^2U\Lambda^{-1/2} - 4k^4\frac{2k^2 - (2n+1)^2U - (2n+1)\Lambda^{1/2}}{k^4 - (2n+1)^2}\Big\}. \tag{3.14}$$

Figure 3 plots the group velocities of the $n = 1$ Rossby wave in the three constant zonal flows. For planetary-scale Rossby waves, the westward group velocity has a larger amplitude in the westerly basic zonal flow and a smaller one in the easterly flow. For smaller scales, while the group velocities are eastward, the effect of the sign of the constant zonal flow is much weaker.

Another significant impact of a constant zonal flow is that the Rossby radius of deformation of the free Rossby wave is modified, as suggested by (3.11b). For the $n = 1$ mode, Fig. 4 shows the Rossby radius, R, increases with k when $U > 0$, remains constant when $U = 0$, but slightly decreases when $U < 0$. For a given mode, R is always greater in the westerlies than in the easterlies, especially for the smaller scales.

The dependence of the Rossby radius of deformation on U has a considerable impact on the meridional structures of the Rossby wave. To illustrate this characteristic, the components and complete structure of the meridional structure function of $n = 1$, $k = 5$ (3.11a) are plotted in Fig. 5. The exponential component of the meridional structure function decays with latitude much faster in the easterly flow than in the westerly flow (Fig. 5a). In Fig. 5b, the Hermite polynomial extends more poleward when $U > 0$ and con-

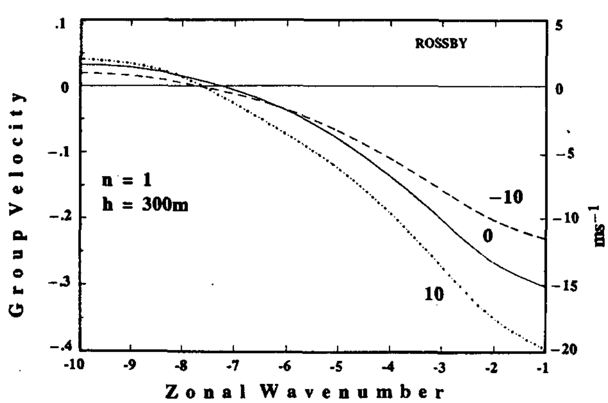

FIG. 3. Group velocity of the $n = 1$ Rossby wave as a function of zonal wavenumber with $U = 10$ (dotted line), 0 (solid line), and -10 (dashed line) m s^{-1}. The left ordinate gives nondimensional value and the right ordinate dimensional value in m s^{-1}.

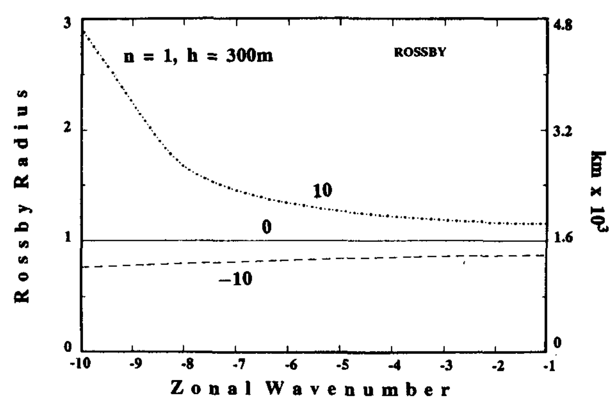

FIG. 4. Rossby radius of deformation, R, of the $n = 1$ Rossby wave as a function of zonal wavenumber with $U = 10$ (dotted line), 0 (solid line), and -10 (dashed line) m s^{-1}. The left ordinate gives the nondimensional value and the right ordinate the dimensional value in 10^3 km.

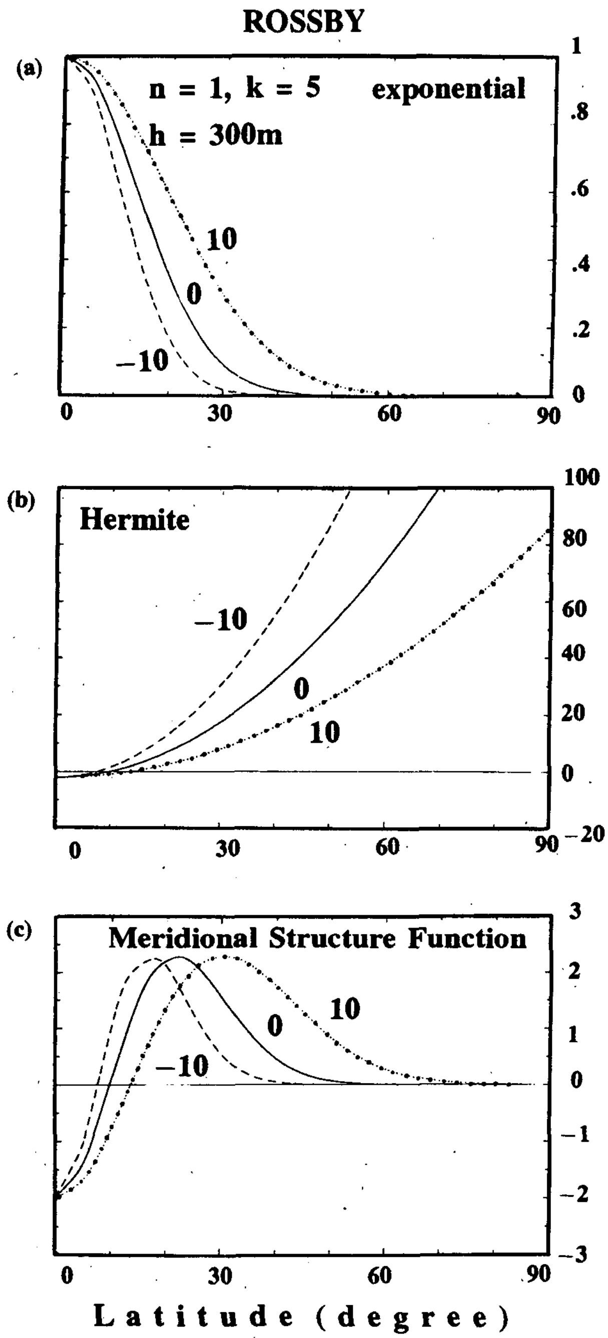

FIG. 5. Meridional distribution of (a) the exponential factor, (b) the Hermite polynomials, and (c) the meridional structure function of the $n = 1$, $k = 5$ Rossby wave with $U = 10$ (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}.

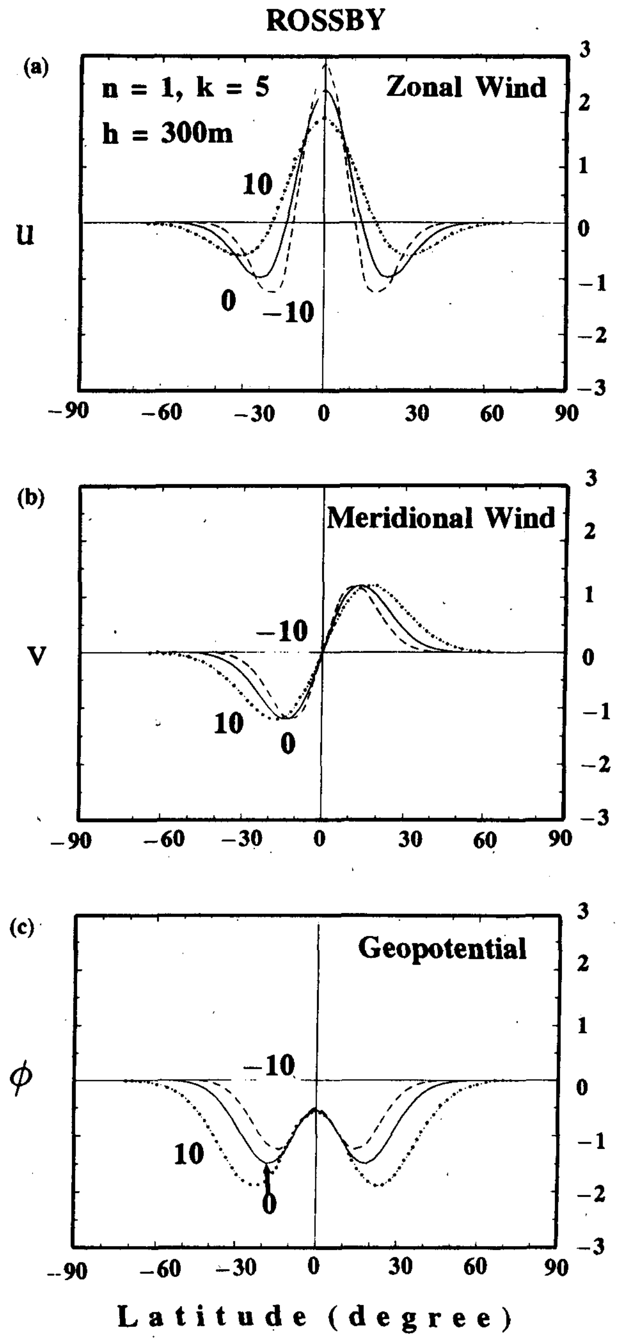

FIG. 6. Meridional distributions of the $k = 5$ Rossby wave solutions with $U = 10$ (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}. Panels show (a) u, $n = 1$; (b) v, $n = 1$; (c) ϕ, $n = 1$; (d) u, $n = 2$; (e) v, $n = 2$; and (f) ϕ, $n = 2$.

tracts more equatorward when $U < 0$ compared to the case with $U = 0$. That is, the Hermite polynomials are displaced either poleward or equatorward depending on the sign of the basic zonal flow. As a result, the complete meridional structure function (Fig. 5c) in westerlies occupies a more extended wave region adjacent to the equator and decays to zero slower toward the poles than in easterlies.

Given the dependencies on constant basic zonal flow of the Rossby radius of deformation, and, especially, the meridional structure function, it is not hard to identify the gross impacts of a constant basic zonal

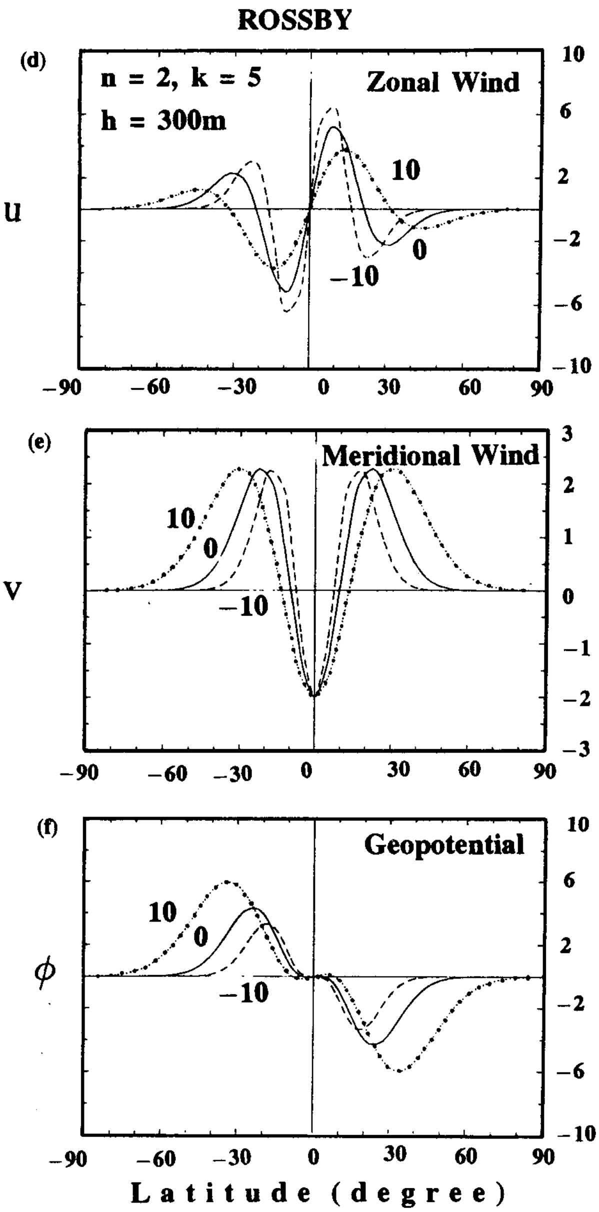

FIG. 6. (*Continued*)

flow on equatorial trapping of the Rossby wave. That is, the equatorial Rossby wave tends to be more trapped in easterlies than in westerlies, and this is particularly so for waves with smaller horizontal scales. The meridional distributions of u, v, and ϕ of $k = 5$ Rossby waves calculated from (3.12) with $n = 1$ and $n = 2$ are plotted in Fig. 6 for the three constant basic zonal flows. The dominant modifications on the meridional structures by the constant basic zonal flow are clearly shown. For a given mode, the basic patterns of these fields are similar regardless of U, but the oscillatory regions extend further poleward from the equator in westerly flow and are more trapped toward the equator in easterly flow. This impact of U on the meridional distributions increases with n. For example, the latitudinal difference between the geopotential maxima in the westerly and easterly flows is 10° for $n = 1$, but increases to about 15° for $n = 2$ (Fig. 6f).

It should be pointed out that the dependence of the structures of the eigensolutions upon U is due to the non-Doppler effect of the basic zonal flow. If only the Doppler effect terms of U are retained in the system [i.e., if term $-yUv$ is neglected from (2.5c)], the characteristics of the corresponding Rossby wave solutions in a constant U will be exactly the same as if $U = 0$ except for ω being replaced by $\hat{\omega}$. Thus, the meridional structures of the solutions, of course, will not be changed by the basic zonal flow. In section 4, we will see that the non-Doppler term modifies the equatorial Rossby wave structure by altering the ambient potential vorticity gradient. The Doppler terms, on the other hand, merely advect the gradient. It also needs to be emphasized that the non-Doppler effect discussed above does not depend upon particular values of nondimensionalization parameters. For example, Fig. 7 shows that the Rossby radius of deformation varies similarly with constant basic zonal flow over a wide range of value of the equivalent depth.

b. Inertia–gravity wave

From (3.9), the approximated dispersion relation for the inertia–gravity wave is

$$\hat{\omega}^2 - k^2 = \left(1 + \frac{k}{\hat{\omega}} U\right)^{1/2}. \qquad (3.15)$$

Generally, $\frac{k}{\hat{\omega}} U \ll 1$ since $\hat{\omega}$ is relatively large for inertia–gravity waves. Consequently, the frequency of the

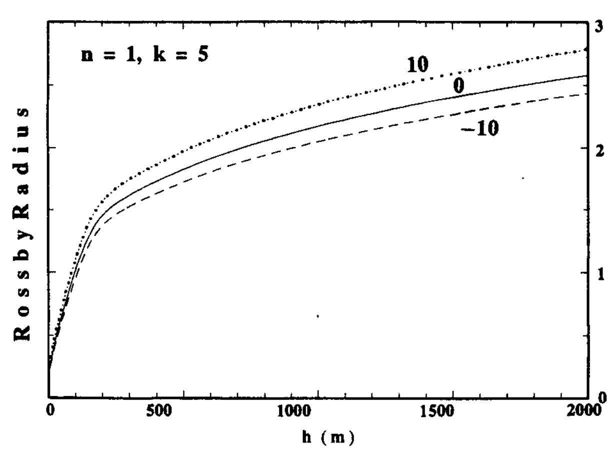

FIG. 7. Rossby radius of deformation, R, of the $n = 1$, $k = 5$ Rossby wave as a function of the equivalent depth with $U = 10$ (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}. The ordinate gives dimensional value in 10^6 m.

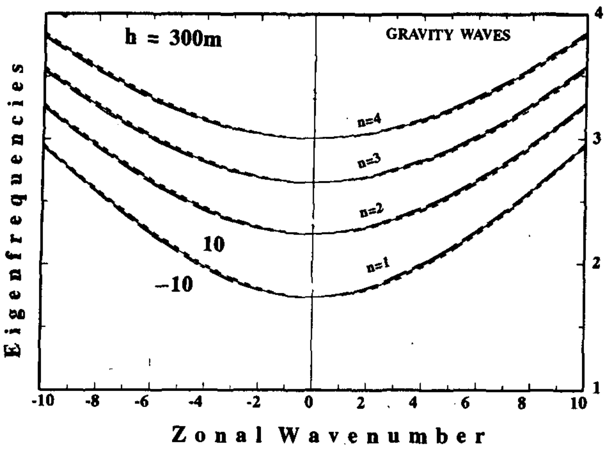

FIG. 8. Dispersion relations of the inertia-gravity waves with U = 10 (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}.

inertia-gravity wave is only negligibly affected by the constant basic zonal flow, as shown in Fig. 8, where $\hat{\omega}$ is plotted as a function of k with n = 1 through 4 for the three basic zonal flows. For the same reason, the Rossby radius of deformation of the inertia-gravity wave and, therefore, its meridional structure given by (3.11b) and (3.12) will not be significantly modified by the constant basic zonal flow.

c. Mixed Rossby-gravity wave

From (3.9), the dispersion relationship for the mixed Rossby-gravity wave is

$$\hat{\omega}^2 - k^2 - \frac{k}{\hat{\omega}} = \left(1 + \frac{k}{\hat{\omega}} U\right)^{1/2}, \qquad (3.16)$$

with the full solutions given by

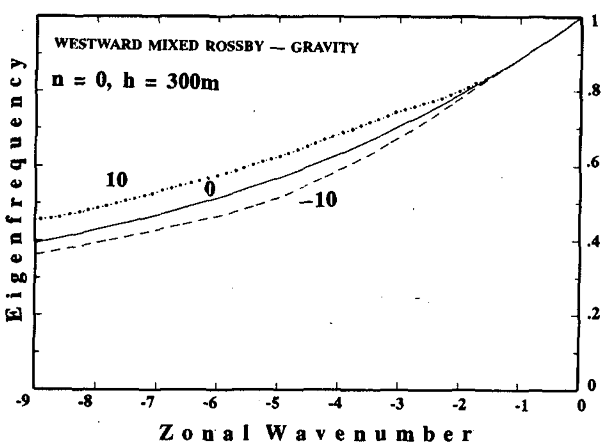

FIG. 9. Dispersion relations of the westward propagating mixed Rossby-gravity wave with U = 10 (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}.

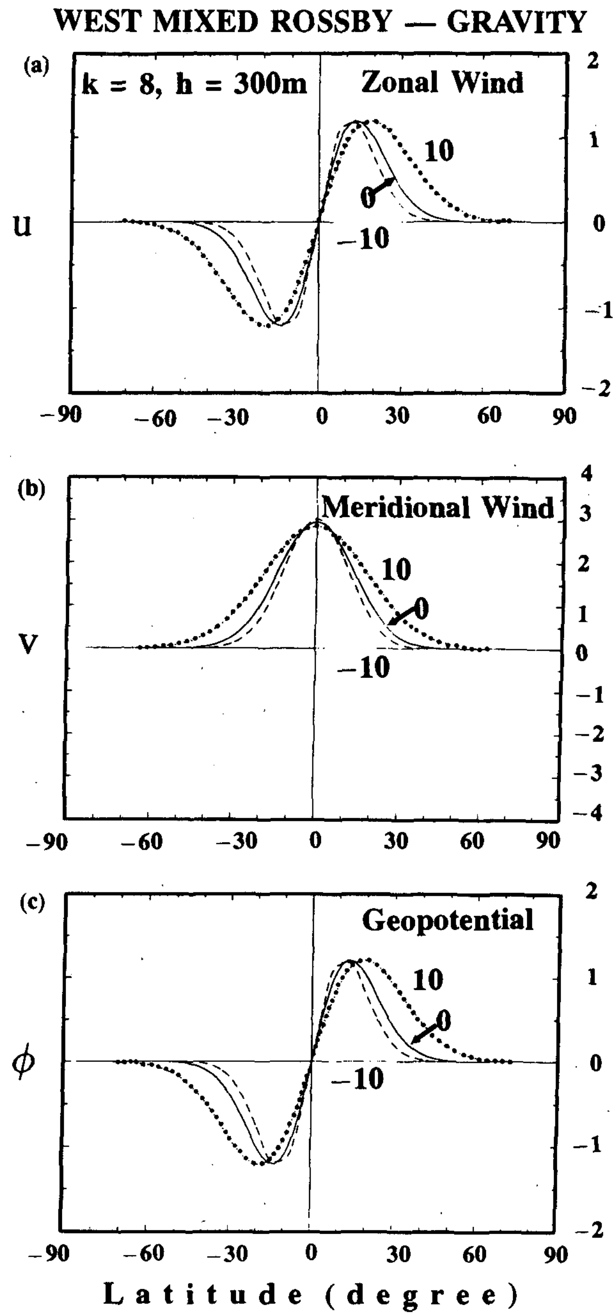

FIG. 10. Meridional distributions of the k = 8 westward propagating mixed Rossby-gravity wave solutions with U = 10 (dotted line), 0 (solid line), and −10 (dashed line) m s^{-1}. Panels show (a) u, (b) v, and (c) ϕ.

$$\begin{pmatrix} u \\ v \\ \phi \end{pmatrix} = \begin{pmatrix} u_1 \mathbf{F}_1(y)/(iW) \\ \mathbf{F}_0(y) \\ \phi_1 \mathbf{F}_1(y)/(iW) \end{pmatrix} \exp[i(kx - \omega t)], \quad (3.17)$$

where $\mathbf{F}_n(y)$, u_1, ϕ_1 and W are defined by (3.11) and (3.12). For the eastward propagating mixed Rossby-gravity wave, the effect of the constant basic zonal flow is negligible because either k is small or $\hat{\omega}$ is large so that

$$\frac{k}{\hat{\omega}} U \ll 1$$

in (3.16). For the westward propagating counterpart, the effect of the constant basic flow remains negligible for small k but becomes larger with increasing k. Figure 9 shows the eigenfrequency of the westward propagating mixed Rossby–gravity wave as a function of k for the three constant basic zonal flows. Like the Rossby wave, larger eigenfrequencies are found in the westerly flow than in the easterly flow. The meridional structures of u, v, and ϕ for the westward propagating mixed Rossby–gravity wave are plotted in Fig. 10 for $k = 8$. Less trapping is found in the westerly flow than in the easterly flow. The similarity to the Rossby wave is not surprising as the westward propagating mixed Rossby–gravity wave asymptotes to a Rossby wave for large k, as seen in Fig. 1 for zero basic state.

d. Kelvin wave

Since the meridional velocity of the Kelvin wave is negligibly small, the term $-yUv$ in (2.5c) vanishes and the non-Doppler effects of the constant basic zonal flow are virtually absent.

4. Potential vorticity interpretations of the equatorial Rossby wave trapping

The physics of the dependence of equatorial Rossby waves upon the basic zonal flow must be addressed in light of the generation mechanism of the Rossby wave. An appropriate physical framework in which to discuss this is the concept of conservation of potential vorticity (PV). As an air parcel moves and experiences changes in ambient PV, perturbation relative vorticity is induced or altered in accordance with the conservation of the total PV. The induced relative vorticity can be viewed as the restoring force that drives the Rossby wave oscillation (Pedlosky 1987, pp. 102–105). In this section, by adopting the prototype conceptual model of an air parcel undergoing a latitudinal displacement relative to the constraint of PV conservation, we shall examine how the form of basic zonal flow affects the restoring force and therefore modifies equatorial trapping of Rossby waves. In other words, we look for the conditions that provide a latitudinal variation in the restoring force defined above. Clearly, an equatorially trapped mode must be driven by a restoring force that decreases with latitude. In an environment that causes the restoring force to decrease rapidly with latitude, a wave will be more trapped and its waveform constrained to lie closer to the equator. A wave that is weakly trapped will have a restoring force decreasing slowly towards the poles and its waveform, forced by the wider distribution of the restoring force, will extend much further poleward. Thus, the degree of the decrease of the restoring forced with latitude signifies the degree of the trapping.

The linear nondimensional PV equation on an equatorial β-plane derived from (2.5) is

$$\frac{d\Pi}{dt} = 0, \tag{4.1}$$

where

$$\Pi = \zeta(1 - \Phi_s) - \phi(\bar{\zeta} + y) + (\bar{\zeta} + y)(1 - \Phi_s) \tag{4.2a}$$

is the total PV. In (4.2a),

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{4.2b}$$

is the perturbation relative vorticity, and

$$\bar{\zeta} = -\frac{dU}{dy} \tag{4.2c}$$

is the relative vorticity of the basic state. With constant U, (4.2a) is simplified as

$$\Pi = \zeta(1 - \Phi_s) - \phi y + y(1 - \Phi_s). \tag{4.2d}$$

Without loosing generality, we assume that initially the perturbation PV of a parcel located at $y = y_0$ is zero. That is, the initial PV is

$$\Pi_0 = y_0(1 - \Phi_{s0}). \tag{4.2e}$$

Then, using (4.1), (4.2d), and (4.2e), the conservation of PV (4.1) states that

$$\zeta(1 - \Phi_s) - \phi y + y(1 - \Phi_s) = y_0(1 - \Phi_{s0}). \tag{4.3}$$

In order to better illustrate how the PV conservation concept can be applied to explain the effects of basic state zonal flow on equatorial Rossby wave trapping, we first consider the situation where the basic zonal flow is zero. For this case, (4.3) becomes

$$\zeta - \phi y + y = y_0,$$

or

$$\zeta - \phi y = -\Delta y, \tag{4.4}$$

where $\Delta y = y - y_0$ represents an infinitesimal meridional displacement experienced by an air parcel. Equation (4.4) states that, under the constraint of PV conservation, for the same displacement Δy, the magnitudes of the induced perturbation relative vorticity, ζ, and geopotential disturbance, ϕ, depend upon latitude. The variation of ζ with y can be obtained if we make an "equatorial quasi-geostrophic approximation." That is, we assume that the geopotential perturbation can be related to streamfunction ψ by

$$\phi = y\psi, \tag{4.5}$$

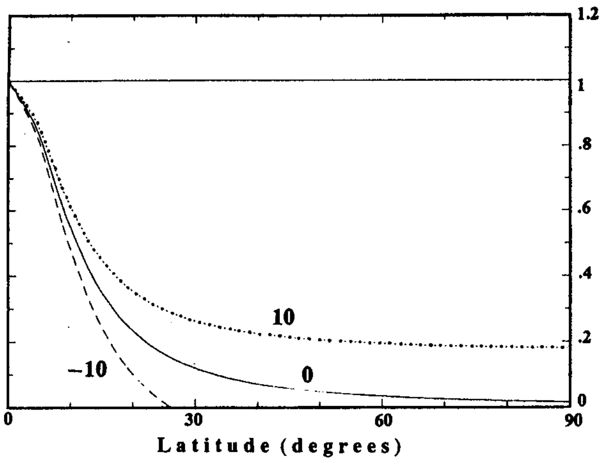

FIG. 11. Meridional variations of magnitude of normalized perturbation relative vorticity with $U = 10$ (dotted line), 0 (solid line), and -10 (dashed line) m s^{-1}. The horizontal line shows the induced perturbation relative vorticity in a nondivergent system.

and the relative vorticity to the streamfunction by

$$\zeta = \nabla^2\psi \approx -\frac{\psi}{L^2}, \qquad (4.6)$$

where L is the horizontal scale of the Rossby wave. With (4.5) and (4.6), the geopotential perturbation ϕ can be expressed in terms of relative vorticity as

$$\phi \approx -yL^2\zeta. \qquad (4.7)$$

The perturbation relative vorticity, then, can be expressed as

$$\zeta \approx -\frac{\Delta y}{1 + L^2y^2}. \qquad (4.8)$$

Equation (4.8) indicates that for the same displacement Δy, the magnitude of the induced perturbation relative vorticity ζ reduces with increasing y from a maximum value at the equator, and therefore, so does the restoring force of the Rossby wave oscillation. The latitudinal variation of ζ for this case is shown in Fig. 11 as the solid line labeled as 0. The decrease of the restoring force with y explains the trapping mechanism of the equatorial Rossby waves.

In a nondivergent system (i.e., $h \to \infty$), the contribution of the geopotential field to the total PV is zero and (4.4) reduces to

$$\zeta = -\Delta y, \qquad (4.9)$$

which shows that ζ is independent of y. In other words, without divergence, the restoring force of Rossby oscillation does not decrease with y so that no equatorial trapping can occur (e.g., see Lim and Chang 1983; Webster and Chang 1988), and an equatorial β-plane is identical to any β-plane on the globe. The PV conservation argument developed here explains physically the critical role divergence plays in the equatorial trapping of Rossby waves.

When a nonzero constant basic state zonal flow is present, an expression of ζ as a function of y can also be derived from (4.3), (4.5), and (4.6) with the geostrophic approximation (i.e., $\Phi_s = -\frac{1}{2}y^2U$). The restoring force then is

$$\zeta = -\Delta y \frac{1 + \frac{3}{2}Uy^2}{1 + L^2y^2 + Uy^2}. \qquad (4.10)$$

Equation (4.10) suggests that while the magnitude of the induced relative vorticity is still inevitably reduced with y in a divergent system, it becomes even more significant in easterly flow ($U < 0$) but less so in westerly flow ($U > 0$). We may then write from (4.10) that

$$|\zeta(U<0)| < |\zeta(U=0)| < |\zeta(U>0)|$$

at any latitude for an equivalent latitudinal displacement. Figure 11 plots the normalized magnitude of ζ from (4.10) against y. The decrease of the amplitude of ζ with latitude is much faster for $U = -10$ m s^{-1} with the e-folding latitude of 15° than for $U = 10$ m s^{-1}, where the e-folding latitude is about 25°. The different decay rates of the restoring force for the different basic zonal flows that the Rossby wave should be more trapped in easterlies than in westerlies, collaborating the results of section 3.

The physical impact of a basic zonal flow on the Rossby wave, which has been described by Pedlosky (1987, pp 108–111) for extratropical Rossby waves, also applies for equatorial Rossby waves. In the presence of a basic zonal flow, the geostrophically balanced geopotential field modifies the latitudinal gradient of the total ambient PV, which is equivalent to an enhanced or reduced β-effect. In a geostrophic westerly regime, the geopotential field tends to decrease with latitude and the β-effect is enhanced. This change in ambient PV leads to a slower decrease with latitude of the restoring force of Rossby wave oscillation and a reduced trapping of the equatorial Rossby waves. On the other hand, in an easterly flow, the effective β-effect is reduced and the change in ambient PV leads to a more rapid decrease of ζ, which provides a stronger trapping of Rossby waves.

5. Meridional solutions in sheared zonal flows

When the basic zonal wind varies latitudinally, the other non-Doppler term $v\,dU/dy$ enters the set (2.5). Equation (2.7) then assumes the form

$$(\Omega - i\omega I)V + iBV = 0, \qquad (5.1a)$$

where

$$\Omega = \begin{pmatrix} 0 & -y & ik \\ y & 0 & \dfrac{d}{dy} \\ ik & \dfrac{d}{dy} & 0 \end{pmatrix}, \tag{5.1b}$$

$$V = \begin{pmatrix} u \\ v \\ \phi \end{pmatrix}, \tag{5.1c}$$

$$B = \begin{pmatrix} kU & -i\dfrac{dU}{dy} & 0 \\ 0 & kU & 0 \\ 0 & iyU & kU \end{pmatrix}, \tag{5.1d}$$

and I is the unit matrix. Let

$$\tilde{V}_m = \begin{pmatrix} \tilde{u}_m \\ i\tilde{v}_m \\ \tilde{\phi}_m \end{pmatrix}, \quad m = 1, 2, \cdots, \infty \tag{5.2}$$

be the mth eigenvector of Ω, and $\tilde{\omega}_m$ the corresponding eigenvalue, such that $\tilde{V}_m$ and $\tilde{\omega}_m$ satisfy

$$(\Omega - i\tilde{\omega}_m I)\tilde{V}_m = 0. \tag{5.3}$$

In other words, $\tilde{V}_m(y)$ is an equatorial normal mode and $\tilde{\omega}_m$ its frequency. It is known that $\tilde{V}_m$ ($m = 1, 2, \cdots, \infty$) can be applied as a basis function for a normal mode expansion (Matsuno 1966). Here we express V in terms of $\tilde{V}_m$ as

$$V(y) = \sum_{m=1}^{\infty} a_m \tilde{V}_m(y), \tag{5.4}$$

where a_m is the expansion coefficient to be determined. By substituting (5.4) into (5.3) with truncation M and using the orthogonality of $\tilde{V}_m$, the following equation is obtained

$$(X - \omega I)A = 0, \tag{5.5}$$

where

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{pmatrix},$$

and X is an $M \times M$ matrix consisting elements $x_{mm'}$ given by

$$x_{mm'} = \tilde{\omega}_m \delta_{mm'} + \frac{1}{N_m{}^2} \int_{-\infty}^{+\infty} (B\tilde{V}_{m'}) \tilde{V}_m^* dy$$

$$= \tilde{\omega}_m \delta_{mm'} + \frac{1}{N_m{}^2} \int_{-\infty}^{+\infty} [kU(\tilde{u}_{m'}\tilde{u}_m + \tilde{v}_{m'}\tilde{v}_m + \tilde{\phi}_{m'}\tilde{\phi}_m) + \frac{dU}{dy}\tilde{v}_{m'}\tilde{u}_m - yU\tilde{v}_{m'}\tilde{\phi}_m]dy. \tag{5.6}$$

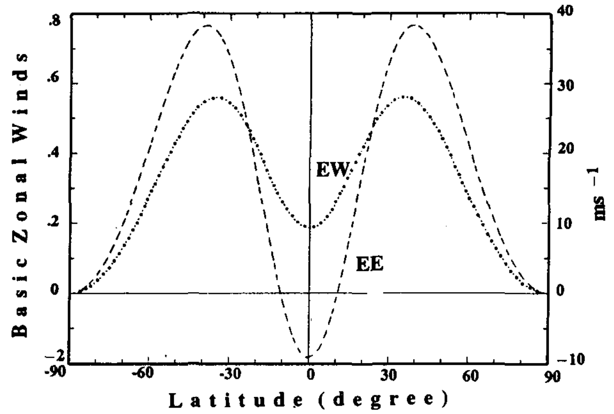

FIG. 12. Basic zonal flows with shears. The flow with equatorial westerly (dotted line) is referred to as EW in the text and the flow with equatorial easterly (dashed line) as EE. The left ordinate gives nondimensional value and the right ordinate dimensional value in m s^{-1}.

In (5.6) $\tilde{V}_m^*$ is the conjugate of $\tilde{V}_m$ and $N_m{}^2$ is

$$N_m{}^2 = \int_{-\infty}^{+\infty} (\tilde{u}_m{}^2 + \tilde{v}_m{}^2 + \tilde{\phi}_m{}^2)dy. \tag{5.7}$$

The infinitive integration in (5.6) and (5.7) can be computed by using the Gaussian–Hermite quadrature formula. Given $U(y)$, X is known, and (5.5) becomes a standard eigenvalue problem that can be solved with A as the eigenvector and ω as the corresponding eigenvalue of X. This method has been described by Boyd (1978a) and employed by Kasahara (1980) to study the effect of zonal flow on the spherical normal modes.

The two basic zonal flows with meridional shear used to calculate X in (5.6) are plotted in Fig. 12. The first (dotted line), referred to as EW, is characterized by an equatorial westerly flow of 10 m s^{-1} and midlatitude westerlies of 30 m s^{-1}. This is a typical meridional distribution of the 200 mb zonal wind over the eastern Pacific Ocean in winter which is shown in Fig. 13. The second flow (dashed line), referred to as EE, resembles the meridional distribution of the 200 mb zonal wind over the western Pacific Ocean in winter (Fig. 13), with an equatorial easterly flow of -10 m s^{-1} and midlatitude westerlies of 40 m s^{-1}.

Using the sheared zonal flows EW and EE, the eigenfrequencies and meridional structures of the equatorial waves are calculated from (5.5) with the truncation including 10 meridional modes (i.e., $M = 33$). With meridional shear at the equator in a basic zonal flow, there is the possibility of inertial instability (e.g., Dunkerton 1981, 1983; Stevens 1983). Also, the modal

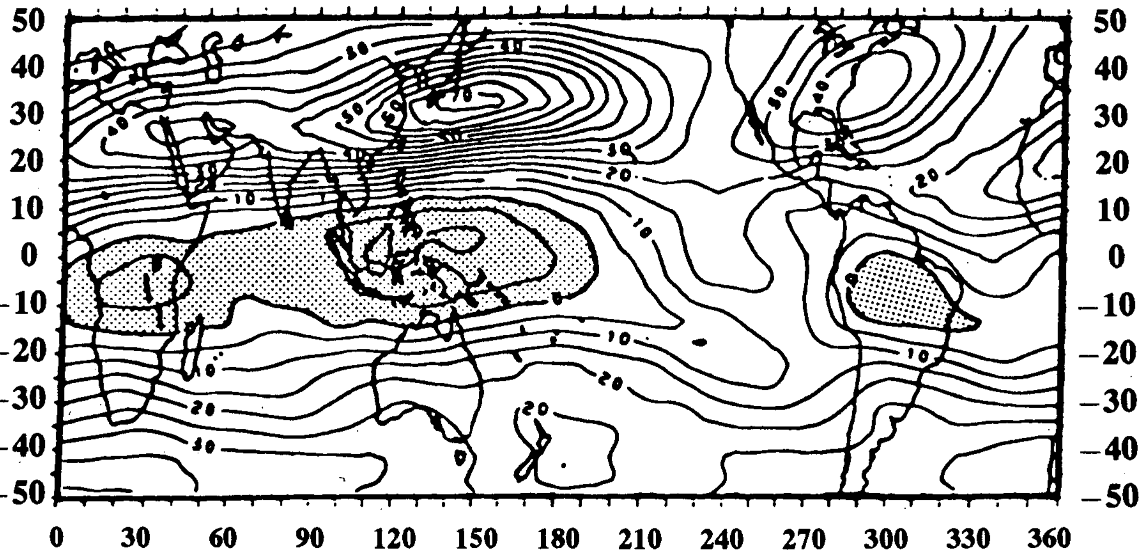

FIG. 13. Observed 200 mb zonal wind in the boreal winter based on a 17-year NMC dataset. Stippled areas denote easterlies. (Contour interval of 5 m s^{-1}).

critical latitude is another issue associated with sheared basic zonal flow that has been addressed by many studies (e.g., Dickinson 1970; Killworth and McIntyre 1985). In our study, all the waves plotted and discussed are inertially stable and no critical latitude exists for those transient waves.

a. Rossby wave

The Rossby wave eigenfrequency is modulated by shear zonal flows in a manner very different from the modification by constant zonal flows discussed in section 3a. Figure 14 shows that the eigenfrequency is always larger in the basic zonal flow EE (dashed line),

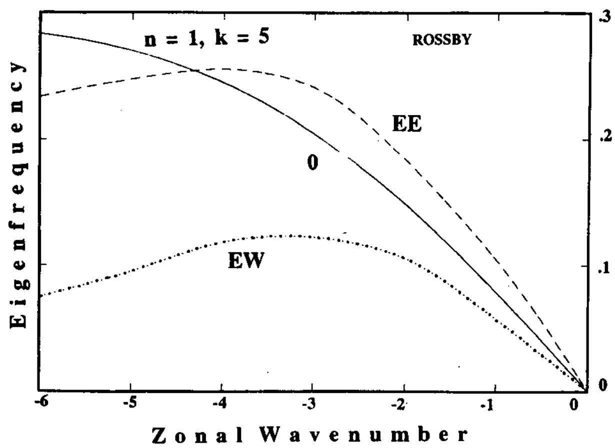

FIG. 14. Dispersion relations of the $n = 1$ Rossby wave in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line).

which contains equatorial easterlies, than in EW (dotted line) which contains equatorial westerlies. For reference, the eigenfrequency for $U = 0$ is also plotted (solid line).

In section 3a, it was seen that the Rossby wave exhibits a larger meridional scale in a constant basic westerly flow than in a constant easterly flow. This characteristic seems to be retained even in basic zonal flows with shears. The meridional structures of u, v, and ϕ for the $n = 1$, $k = 5$ Rossby wave are plotted in Fig. 15. Clearly, the Rossby wave, especially its zonal wind component, is less equatorially trapped in EW than in EE (Fig. 15a). It seems that the meridional structures of the Rossby wave are more controlled by the non-Doppler effects of the basic zonal flows in the equatorial region than those associated with the basic zonal flows of higher latitudes. However, a comparison of the $U = 0$ curve with the EE and EW curves in Fig. 15 indicates that the effect of shear makes the wave generally less trapped about the equator. Perhaps this is consistent with the potential vorticity argument discussed in section 4. Both the basic zonal flows EE and EW enhance the latitudinal geopotential gradient and, therefore, the β-effect between the equator and 35°, the latitude of the westerly jets. The latitudinal decrease of the restoring force of the Rossby wave oscillation in each of the shear flows is then smaller than the case with no shear, and the trapping is reduced.

b. Inertia–gravity wave

In contrast to the situation with constant basic zonal flows, the eigenfrequencies of the inertia–gravity wave are considerably affected by the sheared basic zonal

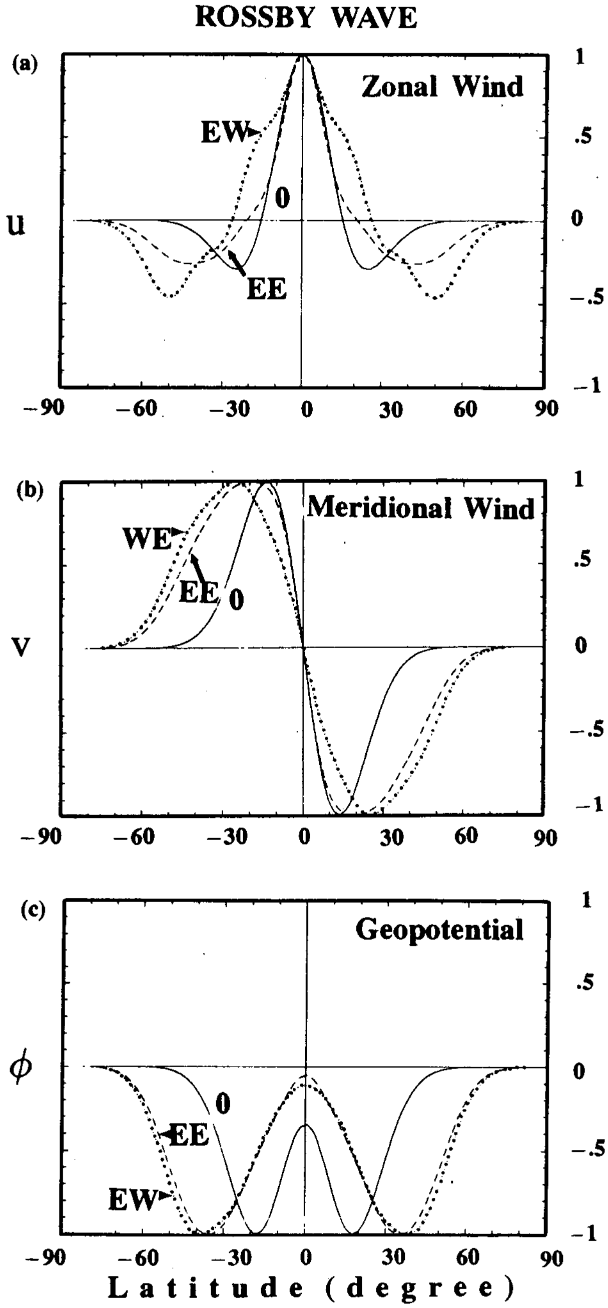

FIG. 15. Meridional distributions of the $n = 1$, $k = 5$ Rossby wave solutions in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u, (b) v and (c) ϕ.

flows. Figure 16 shows that the inertia–gravity wave eigenfrequency in the sheared zonal flows is increased for the eastward propagating waves but decreased for the westward propagating waves relative to the case of zero zonal flow. The eigenfrequency, however, is larger in EW where the shear is weaker than in EE where the shear is stronger for the eastward propagating waves but smaller for the westward propagating waves.

The meridional structures of the eastward propagating inertia–gravity wave, shown in Fig. 17, indicate, however, little differences in the two shear flows with only slightly less trapping occurring in EW than in EE. Figure 18 shows the meridional structure of the westward propagating inertia–gravity wave. Here, the wave appears to be quite sensitive to the sheared flow. Generally, as with the Rossby wave, the trapping is much weaker as a result of shear. However, the wave is even less trapped in EE where shear is stronger than in EW! The implication of this effect on the atmospheric latitudinal interactions is discussed in section 6.

c. *Mixed Rossby–gravity wave*

Figures 19–21 show the effects of shear on mixed Rossby–gravity waves in terms of the eigenfrequency and meridional structures. The effects appear to be similar to those on the inertia–gravity waves in that, for the eastward propagating waves, the eigenfrequencies (Fig. 19) are increased in comparison to the zero basic zonal flow and larger for EW than for EE. The opposite relationship appears for the westward propagating waves. In other words, the meridional structures of the eastward propagating waves seem to be less affected by the shear than those of the westward propagating waves (Figs. 20, 21), at least for u and ϕ. In the sheared zonal flows, the westward propagating wave is much less equatorially trapped in comparison to the zero zonal flow. However, the difference in the meridional structure is not very large between the cases of EE and EW, except for ϕ of the westward propagating wave, which shows obviously less trapped structure in EE than in EW.

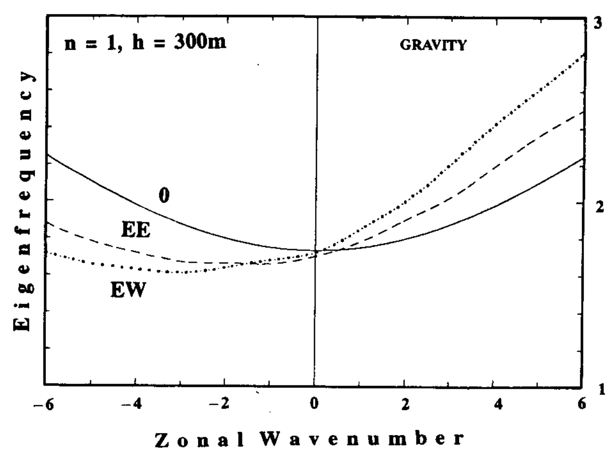

FIG. 16. Dispersion relation of the $n = 1$ inertia–gravity wave in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line).

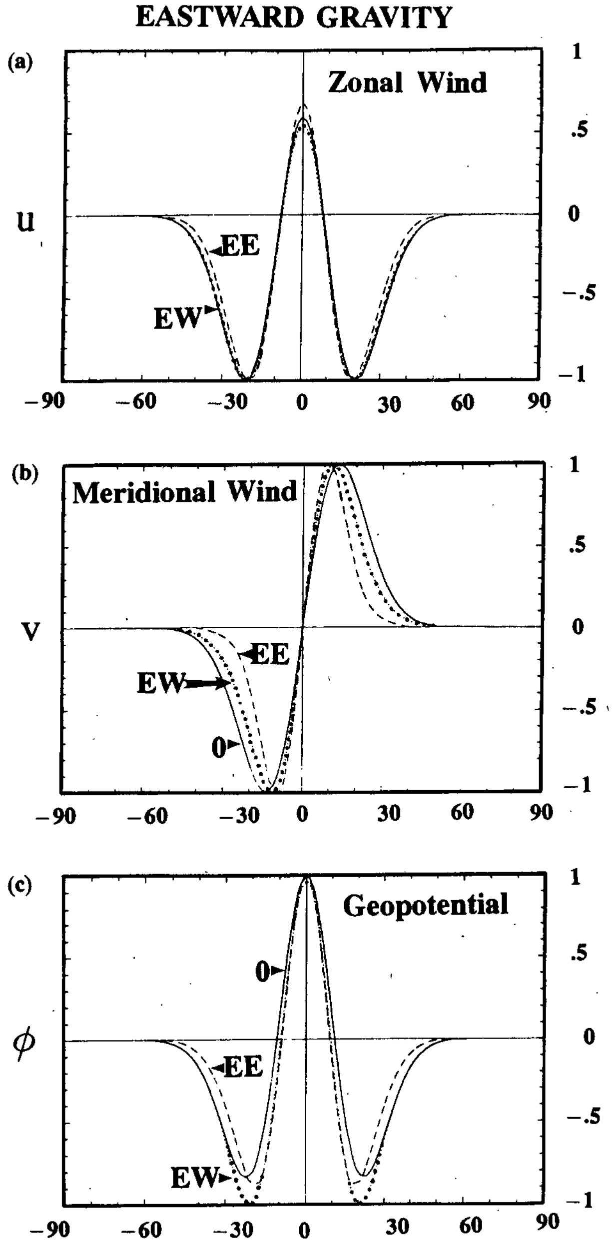

FIG. 17. Meridional distributions of the $n = 1$, $k = 5$ eastward propagating inertia–gravity wave solutions in sheared basic zonal flow EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u, (b) v and (c) ϕ.

d. Kelvin wave

Figures 22 and 23 show the eigenfrequency and the meridional structures of u and ϕ for the $k = 5$ Kelvin wave, respectively. The eigenfrequency is seen clearly to be larger in EW but smaller in EE than in zero zonal flow (Fig. 22). As in a constant basic zonal flow, the meridional structures of the Kelvin wave are basically unaffected by shear (Fig. 23).

6. Summary and conclusions

This study was motivated by some uncertain issues regarding the theory of equatorially trapped waves. These uncertainties were posed by recent observations and theoretical developments. The linear shallow water equations were employed as the framework within which the effects the basic zonal flow on the equatorial waves were addressed. Solutions of the equatorial free normal modes were obtained for constant and sheared

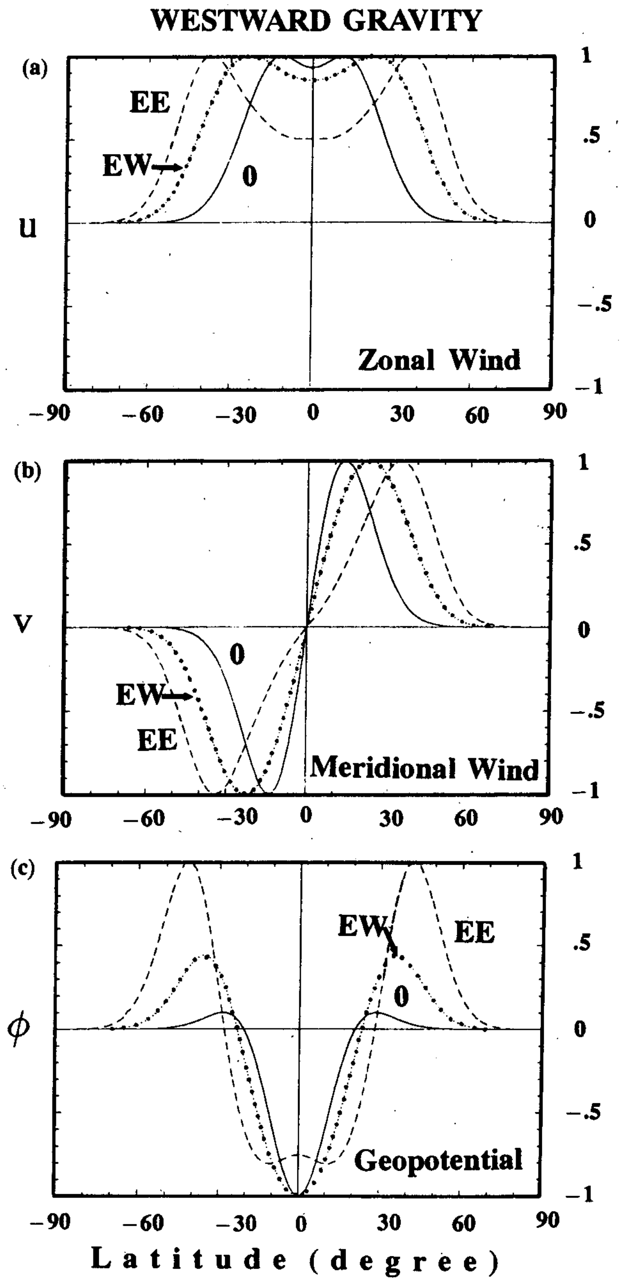

FIG. 18. Meridional distributions of the $n = 1$, $k = 5$ westward propagating inertia–gravity wave solutions in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u, (b) v and (c) ϕ.

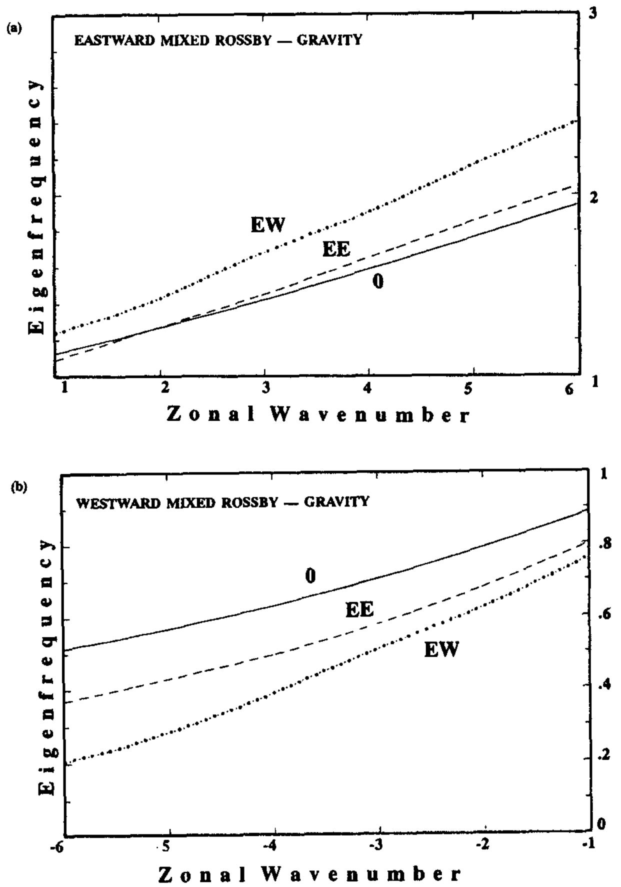

FIG. 19. Dispersion relations of the (a) eastward propagating mixed Rossby-gravity wave and (b) westward propagating mixed Rossby-gravity wave in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line).

basic zonal flows. Most of the results pertain to the $h = 300$ m, $n = 1$ and $k = 5$ waves. However, similar results were also obtained for other parameter values.

The system and the methods employed in this study have their own limitations, of course. The shallow water system restricts addressing the importance of vertical structure of the atmospheric basic state on equatorial waves. The effects of basic meridional flow, which have been shown to be nonnegligible in some circumstances (Schneider and Watterson 1984; Rosenlof et al. 1986), were not considered in this study. Furthermore, it is unknown how the results would be modified by nonlinearity when the amplitudes of the waves become too large for the linear theory to hold. Chang and Webster (1989) suggested, however, that the characteristics of the wave dependence on the forms of the basic state may transcend linearity to nonlinearity. Also, the assumption that the eigenfrequencies and the zonal wavenumbers are constant in sheared basic zonal flows is debatable. Nevertheless, the simple approach adopted here and the major results obtained provide some new insights into the fundamental physics of the equatorial waves. We summarize the main results as follows:

a. General conclusions

(i) The non-Doppler effects of a constant basic zonal flow are significant to the Rossby wave, moderate to the westward propagating mixed Rossby-gravity wave, but negligible to the other equatorial waves. For the Rossby wave and the westward propagating mixed Rossby-gravity wave, the eigenfrequencies were found

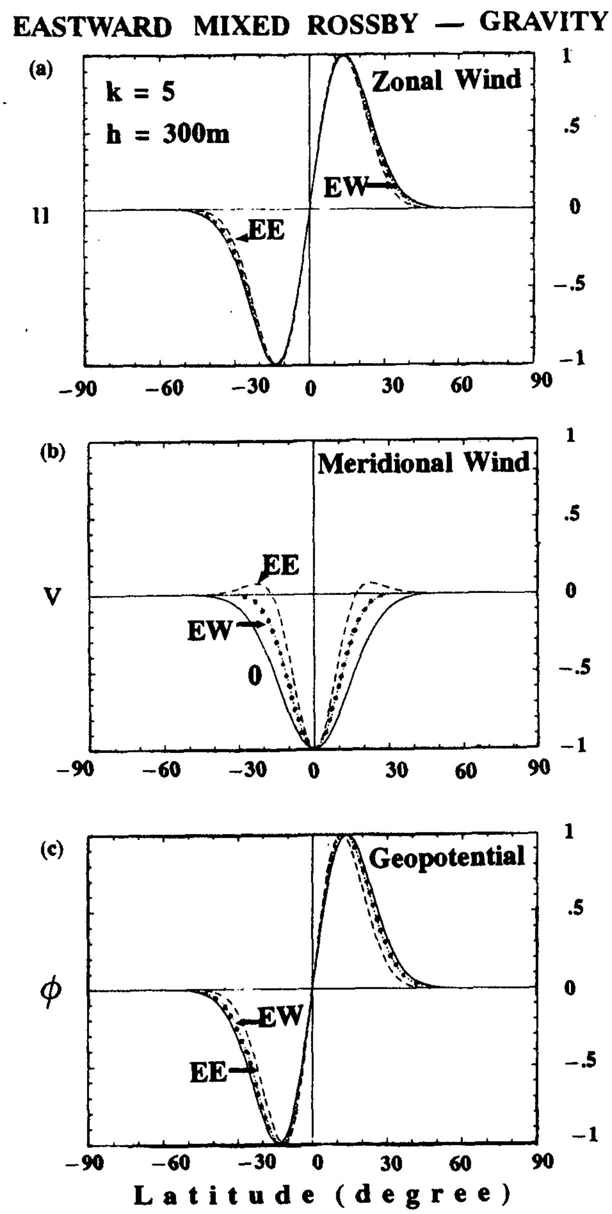

FIG. 20. Meridional distributions of the $k = 5$ eastward propagating mixed Rossby-gravity wave solutions in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u, (b) v and (c) ϕ.

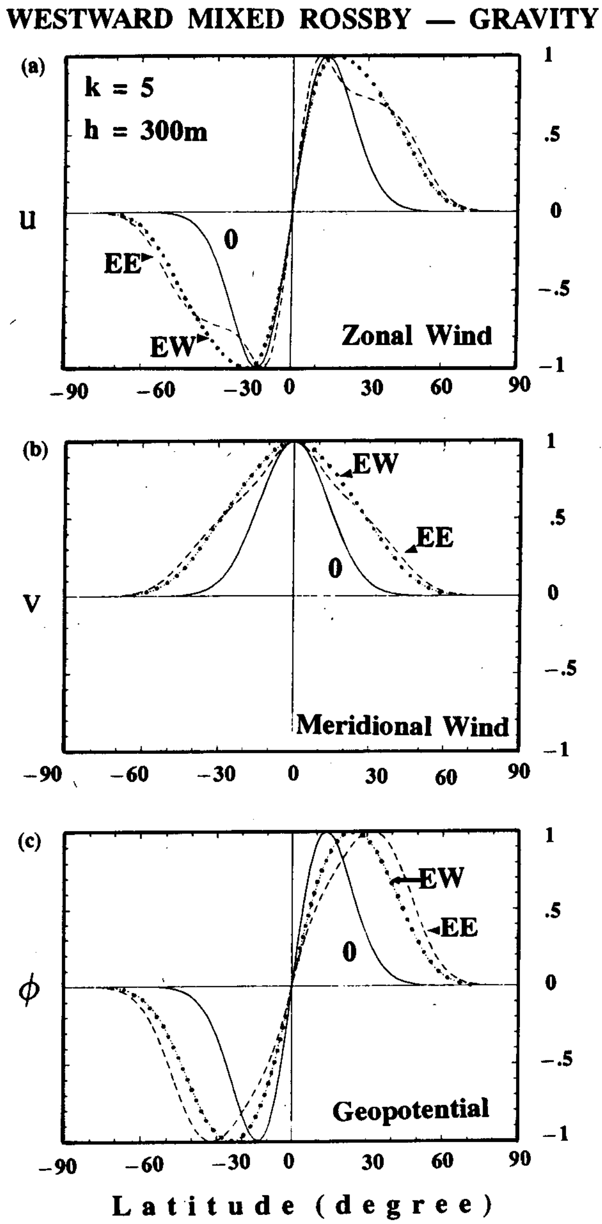

FIG. 21. Meridional distributions of the $k = 5$ westward propagating mixed Rossby–gravity wave solutions in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u, (b) v and (c) ϕ.

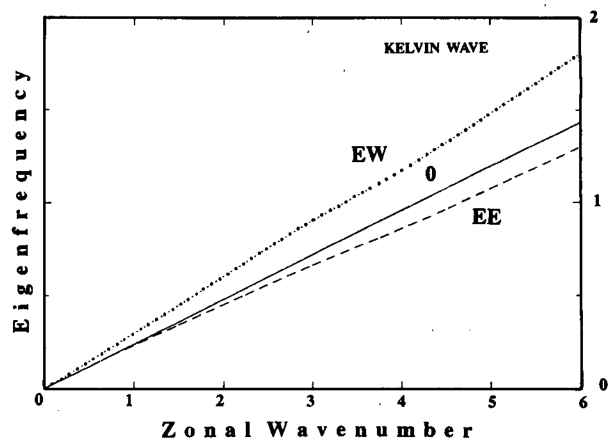

FIG. 22. Dispersion relations of the Kelvin wave in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line).

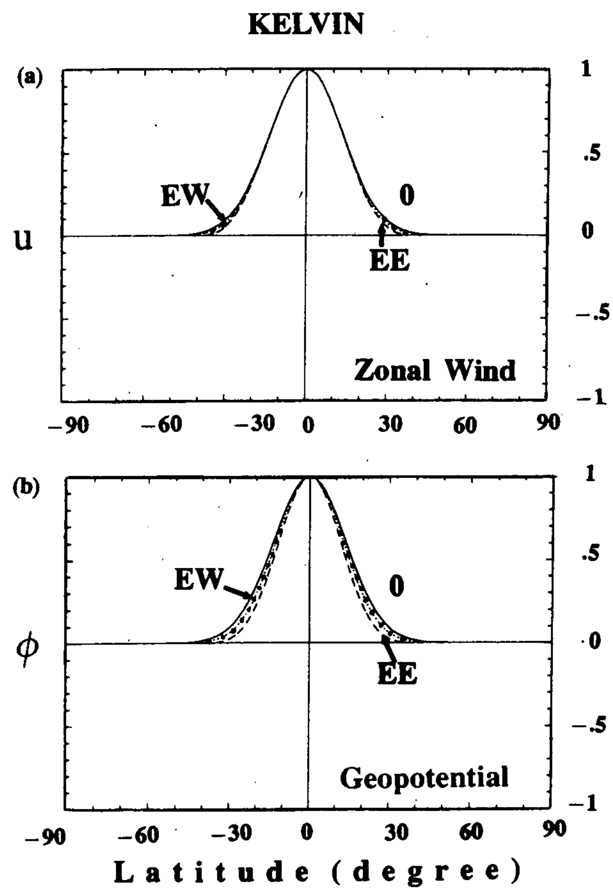

FIG. 23. Meridional distributions of the $k = 5$ Kelvin wave solutions in sheared basic zonal flow; EE (dashed line), EW (dotted line), and in zero basic zonal flow (solid line). Panels show (a) u and (b) ϕ.

to be larger in a constant basic westerly flow and smaller in a constant easterly flow. The meridional structures of these waves were found to be less equatorially trapped in a basic westerly flow than in an easterly flow.

(ii) The non-Doppler effect of a constant basic zonal flow on the Rossby wave trapping was interpreted in terms of potential vorticity conservation. Basically, in a divergent system with potential vorticity conservation, the induced perturbation relative vorticity (i.e., restoring force of the Rossby wave oscillation) reduces with latitude at different rates depending upon the sign of the basic zonal flow and produces different degrees of equatorial trapping. Westerly zonal winds enhance

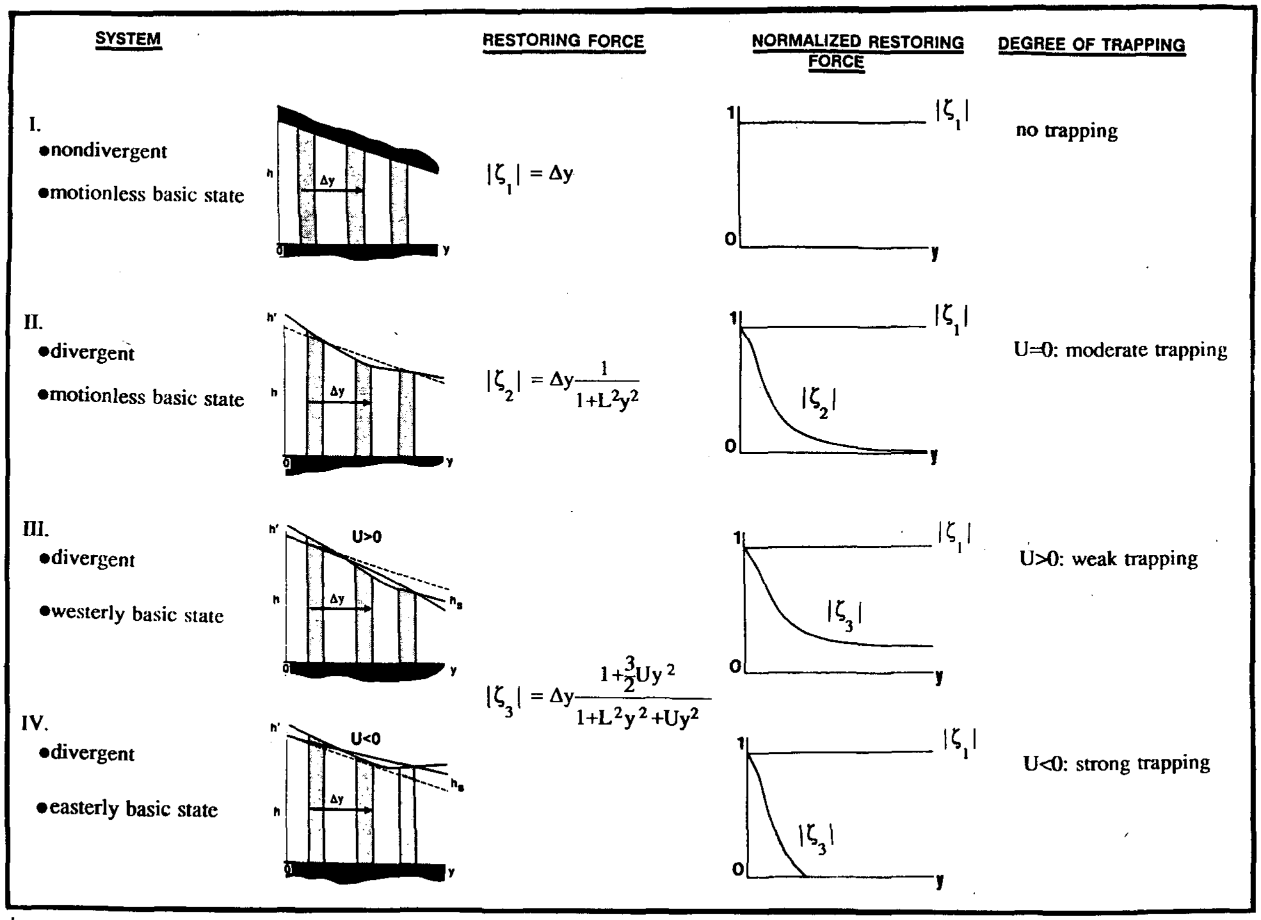

FIG. 24. Summary diagram of the physical processes involved in determining the equatorial trapping of atmospheric Rossby waves. Diagram refers to the relationship between the induction of perturbation relative vorticity and the Rossby wave forcing mechanism by latitudinal displacements in different basic zonal flows. See text for more details.

the ambient potential vorticity gradient and lead to a slower reduction of the restoring force with latitude and thus weaker trapping. Through the opposite effect, the reduction of the ambient potential vorticity gradient, easterly zonal winds lead to a substantially stronger trapping. A summary diagram of the potential vorticity interpretation is given in Fig. 24. The diagram shows four physical systems (I through IV), the characteristics of which are listed in the left column. The second column shows latitudinal cross sections of the "equivalent" shallow fluid systems. System I is nondivergent with variable top slopes signifying the rotational variability with latitude. Systems II through IV are three surface systems that are divergent (h' denotes the free surface deviation indicated by the curved solid lines). The dashed lines in Systems II through IV refer to the solid surface deviations with latitudes that emulate the β-effect. The free surface deviation h_s (straight solid line in Systems III and IV) is the slope required to geostrophically balance the zonal flow U. The shaded vertical bars denote vortex tubes undergoing a northward displacement Δy. The third column lists the restoring forces resulting from the latitudinal displacements for $U = 10, 0, -10$ m s^{-1}, respectively. The distributions of the normalized restoring forces (i.e., $|\zeta|/\Delta y$) are shown in the fourth column and the extent of trapping is described in the last column. Of particular interest is the difference in the degree of trapping between Systems III and IV.

(iii) The effects of shear on the equatorial waves are quite complicated. In general, the eigenfrequencies were found to be larger in the sheared basic zonal flows with equatorial easterlies than in those with equatorial westerlies for the westward propagating waves, but smaller for the eastward propagating waves. The meridional structures of the eastward propagating waves are generally not significantly affected by the sheared basic zonal flow, while the westward propagating waves were found to be considerably less equatorially trapped than those in a zero zonal flow.

In a qualitative sense at least, the general decrease in the Rossby wave trapping in shear flow can also be interpreted in terms of the potential vorticity conservation. Even though the values of the sheared zonal flow EE and EW in Fig. 12 are of different sign at low latitudes, both possess strong positive values of dU/dy between the equator and the latitudes of the extratropical westerly jet maxima. In terms of the equivalent system discussed in (ii), the free surface height must reduce rapidly with increasing latitude to support this shear. This has the effect of enhancing the β-effect even further. Consequently, the induced perturbation relative vorticity (i.e., the restoring force of the Rossby wave oscillation) reduces even more slowly with latitude, resulting in a reduction in trapping.

(iv) Returning to Fig. 13, we note that regions of equatorial easterlies are at longitudes where strong latitudinal shear exists. Equatorial westerly regions, on the other hand, correspond to regions of weak shear. For the Rossby wave, the combination of the sign of the basic zonal flow at low latitudes and the impact of shear still acts in the same direction as in the cases of constant basic zonal flows. That is, those waves whose zonal scales are smaller than the longitudinal variation of the basic zonal flow will be much more trapped in the eastern hemisphere than in the western hemisphere. On the other hand, the westward propagating inertia-gravity wave exhibits less trapped structures in easterly basic zonal flows than when the flows are westerlies.

(v) The dependence of the westward propagating equatorial waves upon the basic zonal flow suggests that the variations in the atmospheric basic state must be considered to explain the observed differences in the wave characteristics, such as those discussed in section 1. For example, in Fig. 19, $\hat{\omega}$ varies with k more rapidly for the basic zonal flow EW (dotted line) than for EE (dashed line), implying that the group velocity of the westward propagating mixed Rossby-gravity wave should be faster in equatorial westerlies than in equatorial easterlies. This difference has been observed by Liebmann and Henden (1989) in the lower tropical troposphere.

b. *Implications to latitudinal interactions*

(i) The dependence of the degree of equatorial wave trapping on the sign and shear of the basic zonal flows is of extreme importance if the dynamics of tropical–extratropical interactions are to be understood. A common concept is that in steady-state or stationary regimes, the easterly regions of the deep tropics are insulated from the extratropics by critical latitudes (e.g., Webster and Holton 1982; Nigam and Held 1983). On the time scale of tropical convection, however, most of the energy generated by latent heat release is carried by transient inertia-gravity waves (Schubert et al. 1980). The results from this study indicate that direct interactions between tropical easterly regions and extratropics are possible through the westward propagating inertia-gravity wave since it is less equatorially trapped within a sheared basic zonal flow with equatorial easterlies (e.g., EE in Fig. 18) and not affected by a critical latitude. The possible impacts of the inertia-gravity wave generated in the tropics on extratropical large-scale motions have been reviewed by Paegle et al. (1983).

(ii) On time scale for which the Rossby wave dominates (e.g., the synoptic and low-frequency time scales), the scenario of tropical-extratropical interaction is quite different. It has been shown that stationary waves are not capable of propagating in and out of

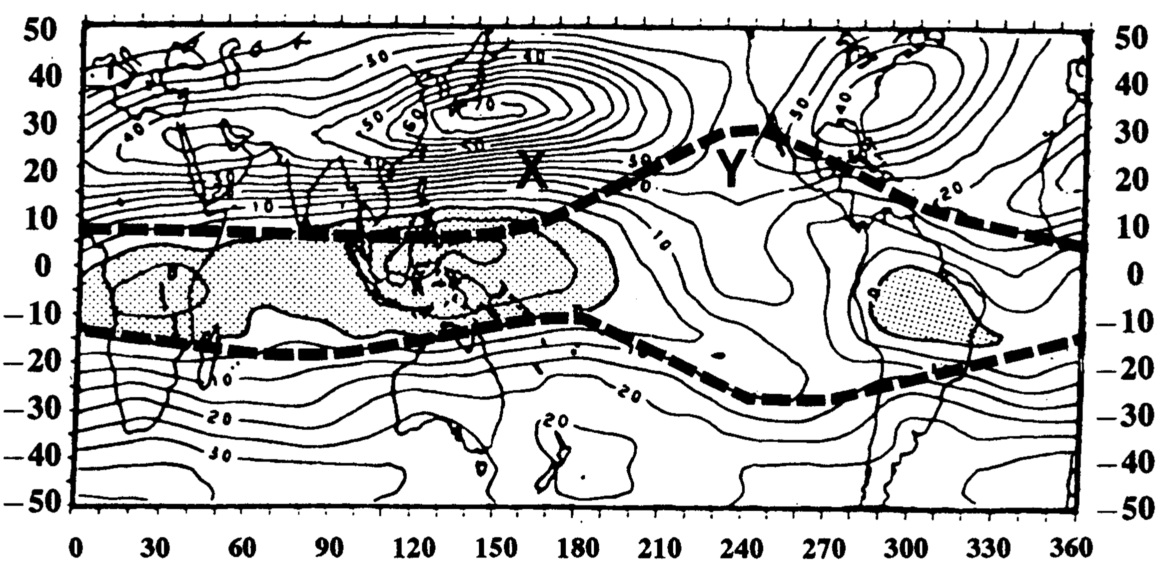

FIG. 25. The distribution of the turning latitude of an equatorially trapped Rossby wave as a function of longitude and latitude in the two-dimensional basic boreal winter 200 mb zonal wind shown in Fig. 11. Letters X and Y denote forcing locations. Forcing located at X where $y > y_t$ will not project significantly on equatorial trapped waves. Forcing located at Y where $y < y_t$ will force trapped waves.

tropical easterly regions because of the existence of critical latitudes. However, they are free to propagate through the tropical westerly areas (Webster and Holton 1982). In this study we have shown that transient Rossby waves are less trapped in equatorial westerlies than in equatorial easterlies. Therefore, it appears that the tropical–extratropical interactions through Rossby waves on a wide time scale are more likely to occur at longitudes where the tropical basic zonal winds are westerlies.

The consequences of these conclusions with respect to weather and climate are summarized in the schematic diagram Fig. 25. The diagram shows qualitatively the variation of a Rossby wave turning latitude (dashed lines) within a boreal winter basic zonal flow at 200 mb. A strong longitudinal variation depending upon the sign of the basic zonal flow and the degree of latitudinal shear is apparent. Regions exist where the equatorial Rossby wave extends well into the extratropics and others where it is constrained laterally about the equator. Clearly, the variation of the basic state has a dominant influence on the modal structure.

We feel justified in making the following speculations that relate directly to the interactions of remote circulation regimes. These are:

1) Webster and Chang (1988) suggested that transient Rossby waves emanate from their energy accumulation region within the upper tropospheric westerlies. The results from this study allow another interpretation of their "emanation." Rather than the production of a Rossby wave train, in which the equatorial westerlies act as a wave train source, the influence on the extratropics may better be thought of as a "waveswelling" since equatorial Rossby waves propagating along the equator into the westerlies increase their turning latitudes substantially and, in this sense, extend their influences into the extratropics. It should be remembered, however, that these waves are longitudinally trapped by the stretching deformation of the basic zonal flow to the west of the region where the lateral influence takes place.

2) Since the wave regions of certain equatorially trapped Rossby waves can extend well into the extratropics, midlatitude forcing sources may project onto equatorial waves and thus excite trapped waves along the equator. It is thus conceivable that midlatitude influences can extend into the easterly regimes of the tropics, but via the generation of trapped waves in the westerlies which then propagate along the equator. For example, the forcing source located near X in Fig. 25, and thus at a latitude where $y > y_t$, will not project significantly onto equatorially trapped waves. On the other hand, forcings located near Y where $y < y_t$ can invoke trapped responses. This process is a refinement of the Webster–Holton hypothesis, which saw the midlatitude influencing the tropics in the steady-state regime if the wave sources were placed along ray paths that led into the "westerly duct" (Webster and Holton 1982).

The two speculations discussed in 1) and 2) indicate the importance of the westerly regions of the upper troposphere, in general, and the Pacific Ocean, in particular. On interannual time scales the dynamics of the coupled ocean–atmosphere system of the Pacific Ocean appear to dominate. On a much broader time span, the atmosphere over the eastern Pacific Ocean appears as a clear corridor for the interaction of waves either generated in the extratropics or in the tropics. More formally, it may be stated that the eastern tropical Pacific Ocean is a region where the influence regions of a broad range of waves, normally geographically isolated from each other, tend to overlap.

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