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Supporting Online Material for

Deconvolution of the Factors Contributing to the Increase in Global Hurricane Intensity

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Supplementary Material

Mutual information is based on the concept of entropy, which is associated with the randomness in a signal. Mutual information measures, in bits, the independence of the two variables. Claude E. Shannon (S1) originally defined entropy, H(X), for a signal *X* as

$$H(X) = \sum_{x} f(x) \times \log_2 f(x)$$

where a random event x occurs with a probability $f(x)$. The joint entropy of two variables *X* and *Y* measures the entropy contained in the joint system and is defined as

$$H(X,Y) = \sum_{x,y} p(x,y) \times \log_2 p(x,y)$$

If *X* and *Y* are independent, the the total entropy of the system would be equal to $H(X) + H(Y)$. In all cases, $H(X) \leq H(X,Y)$, and the equality is only achieved when *X* and *Y* are totally dependent. Based on these definitions, mutual information can be defined as

$$I(X,Y) = \sum_{x,y} p(x,y) \times \log_2 \frac{p(x,y)}{f(x)g(y)}$$

or as

$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$

To illustrate these concepts, let *X* be a normally distributed random variable $X \sim N(0,1)$ and *Y* be defined as $Y = (B \cdot X + C \cdot Z)/K$, where *B* and *C* are constants, $Z \sim N(0,1)$ and *K* is a scaling factor ($K = \sqrt{B^2 + C^2}$), so $Y \sim N(0,1)$. Note that for $B = 0$ and $C = 1$, *X* and *Y* are totally independent. Conversely, if $B \neq 0$ and $C = 0$, *X* and *Y* are absolutely dependent. Figure S1a shows the discrete marginal probability density function (PDF) of

X and Y after generating 500.000 random numbers which are grouped in a 30-bin histogram. In this setup, the entropy of X and Y is 3.70 and hence the mutual information varies from 0 (total independence) to 3.70 (total dependence). Figure S1b and Figure S1c show the joint distribution of X and Y for $C = 1$ and $B = 0$ and $B = 0.5$ respectively. In the first case the joint distribution is equal to the product of the marginal distributions and the mutual information is zero. In the second case, the mutual information is 1.11, implying that by knowing X, a fraction of Y is also known. As B increases, the mutual information of X and Y goes to zero, implying a loss of dependence. Figure S1d shows results for $B = 1$.

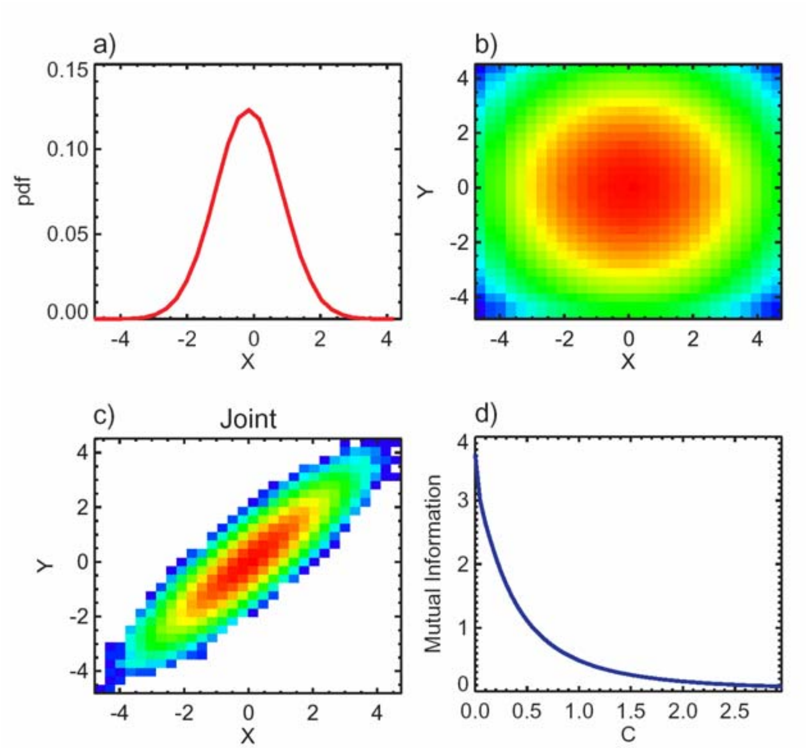

Figure S1. a) Discrete marginal probability density function (PDF) of X and Y. b) and c) Joint distribution of X and Y for $C = 1$ and $B = 0$ and $B = 0.5$ respectively. d) Change of mutual information relative to C for $B = 1$.

Now, assume that we have two related variables X and Y defined as following

$$X = X_1 + X_2$$
$$Y = K \cdot X_1 + L \cdot X_2 + Y_1$$

where K and L are constants, $X_1 = a + b \cdot t$ represents a trend in the signal X (Fig. S2a), $X_2 = c \cdot \sin(\omega \cdot t)$ represents the variability of X, and $Y_1 \sim N(0, \sigma^2)$. According to the definition of Y, the relative magnitude of K and L determine the degree of dependence of Y on X. If $K = 1$ and $L = 0$ (Y_a; Fig. S2b), the dependence is exclusively due to the trend; if $K = 0$ and $L = 1$ (Y_b; Fig. S2c), then the dependence is due to variabilty.

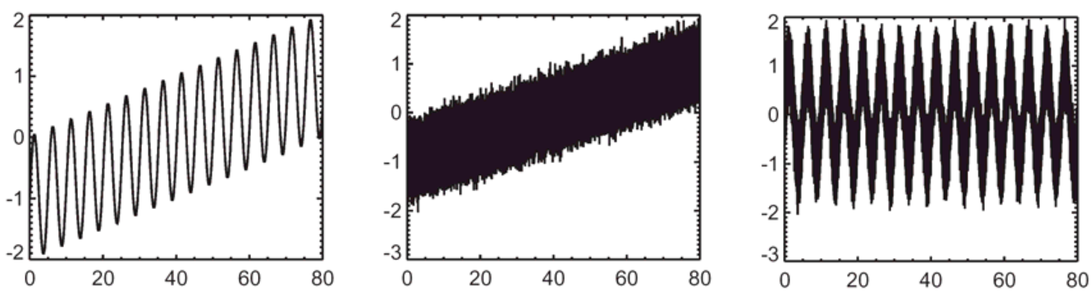

Figure S2. Variables a) X, b) Y_a and c) Y_b.

When variables that share information are composed of different signals and scales of variability, it is vital to determine the origin of the mutual information. That is the situation for X-Y_a (case A) and X-Y_b (case B). In case A, we know that the information shared corresponds to the trend in both variables, while in case B, the relationship between the variables is the periodic variability. Here we illustrate for the synthetic cases A and B the mutual information analysis by isolating the components of the variables and evaluating the source of the information shared. Figure S3 shows the scaled joint distribution ($p(x,y)/f(x)g(y)$) of X and both versions of Y (Y_a and Y_b) as well as the scaled joint distributions isolating the trend and the variability in X (X_1 and X_2). Grids with values greater than one in the scaled joint distribution contain the information shared by the variables. The scaled distributions of both X-Y_a (Fig. S3a) and X-Y_b (Fig. S3d) illustrate the expected proportional relatioship between variables. The mutual information in both cases is 0.42 and 0.57 respectively, compared to the maximum achievable mutual information of 4.32 and 4.39. The mutual information is relatively low due to the effects of the information not shared by the variables: variability in X and randomness in Y_a (case A), and trend in X and randomness in Y_b (case B). When the trend in X is isolated from the variability, the mutual information of X and Y_a (case A) increases to 1.27, which is shown graphically by Fig. S3b. Once again, information shared is not the maximum value due to the randomness in Y_a. In contrast, the mutual information of X and Y_b (case B) decreases to 0.28; in Fig. S3e most values are less than one. On the other hand, when the trend is removed, the information between X and Y_a (case A) decreases to 0.01 and between X and Y_b (case B) increases to 1.41. Figures S3c and S3f explain graphically the mutual information for cases A and B after removing the trend. In summary, the mutual information and the scaled joint distribution successfully shown whether the information shared by two variables in cases A and B results from the trend or from the periodic variability.

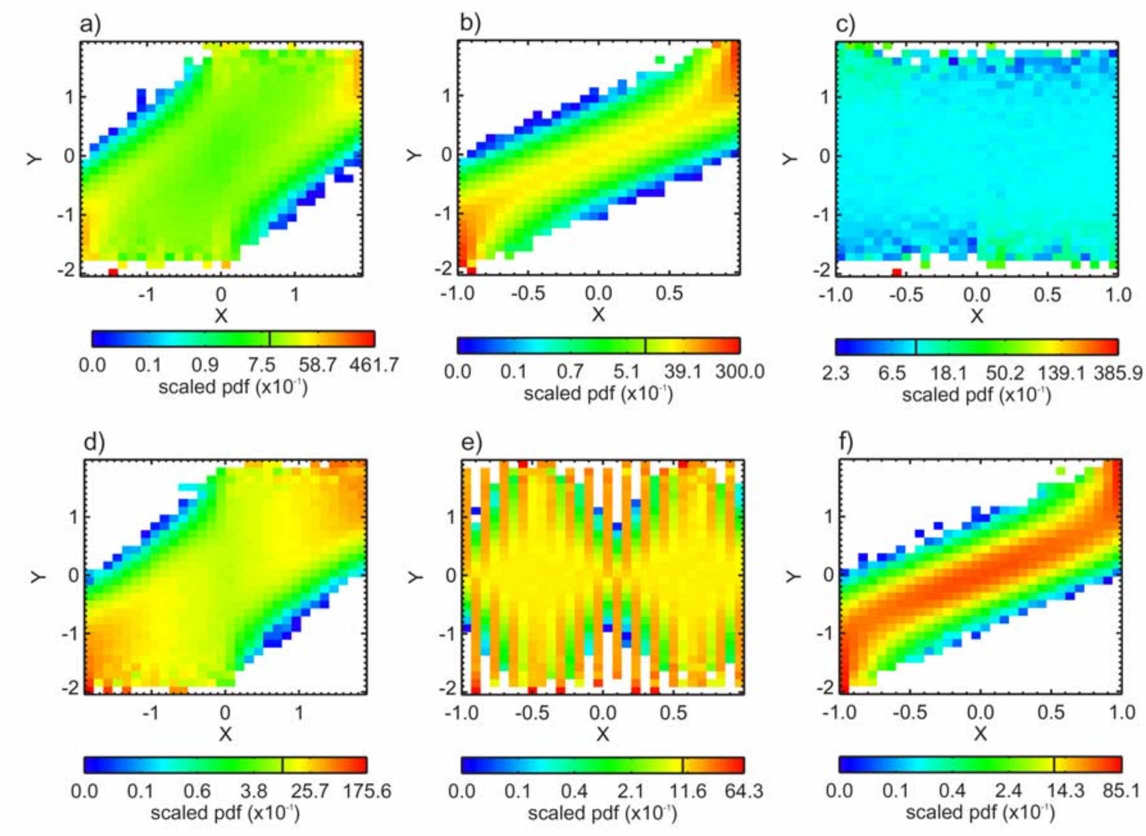

Figure S3. Scaled joint distribution of a) X and Y_a, b) X_1 and Y_a, c) X_2 and Y_a, d) X and Y_b; e) X_1 and Y_b; and f) X_2 and Y_b. Vertical line in the color bar corresponds to one.

References

S1. C. E. Shannon, *Bell System Tech. J.* **27**, 379 (1948).

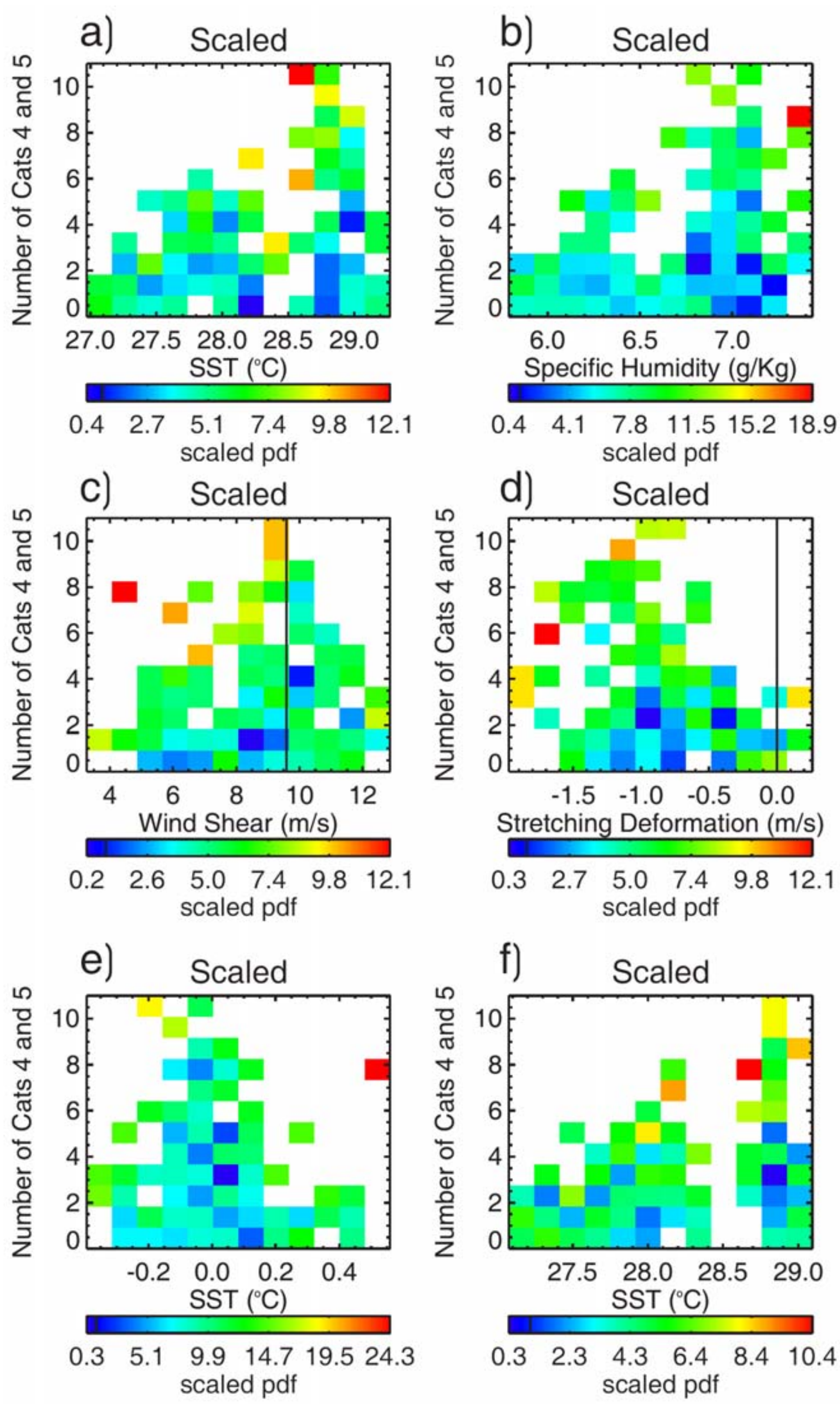

Figure S4. Scaled distribution in all basins except NIO for a) SST, b) Specific Humidity, c) Wind Shear, d) Stretching Deformation at 850 mb, e) SST variability, and f) SST trend. Results do not change substantially when compared to those obtained using all six basins (Figures 2- 4).

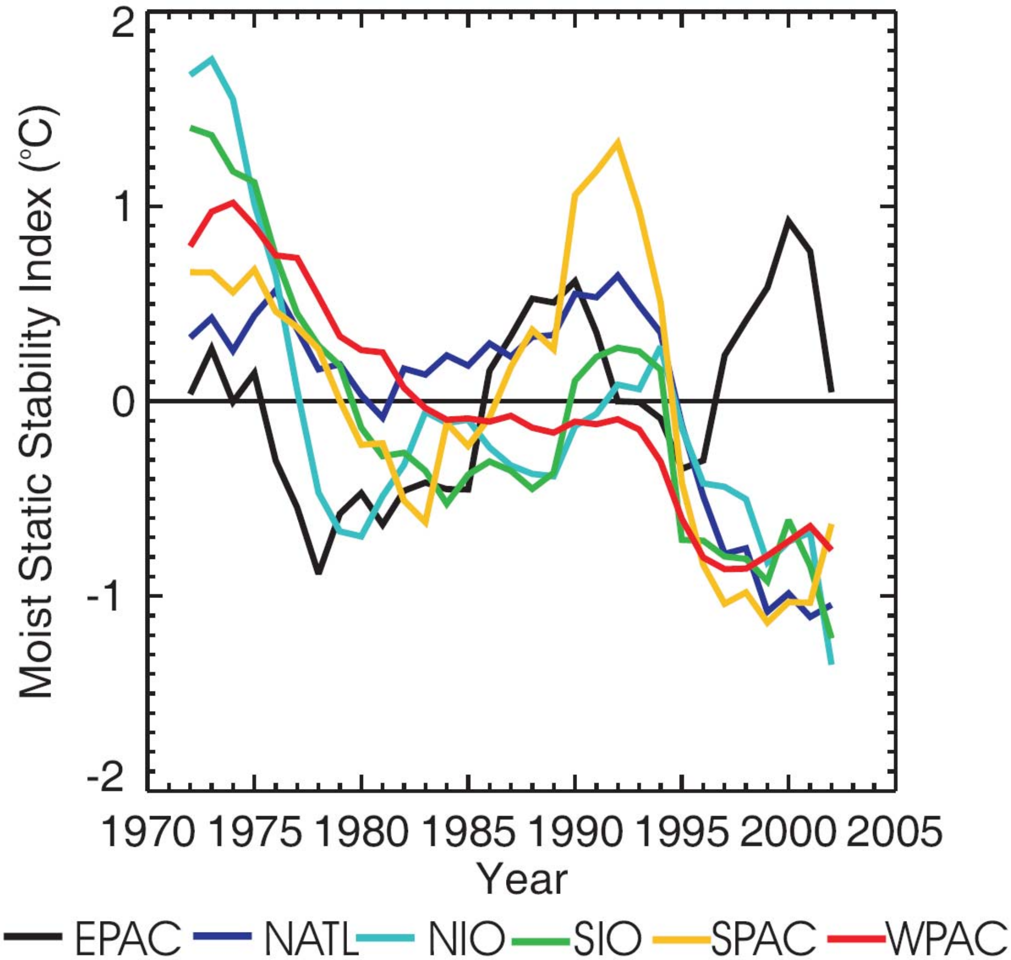

Figure S5. 5-year Moving average anomalies relative to the 1970-2004 period of Moist Static Stability Index. The index is defined as the difference between the equivalent potential temperature at 500mb and 1000mb. The standardized trends of the moist static stability (comparable to those in Table 1) for all the basins are: EPAC 0.82, NATL **-2.14**, NIO **-2.33**, SIO **-2.62**, SPAC **-1.88**, and WPAC **-5.68**. Values in bold are statistically significant at the 99% confidence level.